Index Mutual Fund Replication

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Motivation of Index Fund Tracking I

- The literature shows that most of actively managed funds usually do not outperform index mutual funds which are passively managed to mimic certain indices in the long-term (see [Malkiel, 1995], [Carhart, 1997], [Bogle, 1999] and [Haslem et al., 2008]).
- The institutional investment in index funds was increased dramatically after the bankruptcy of major investment banks on Wall Street in September 2008 (Investment Company Institute).

Motivation of Index Fund Tracking II

- Fund expenses, such as management fees or distribution fees have continued to increase during the last two decades. The question of whether charging higher fund fees benefits investors has been discussed at great length (see [Anderson and Ahmed, 2005]). Researchers have drawn attention to a confusing phenomenon in the fund market: while fund fees and expenses vary quite a lot, the return patterns of the funds typically show relatively small amounts of dispersion.
- There are many papers about index tracking; however, using index tracking techniques to replicate index mutual funds has not been widely discussed in the literature yet.

Motivation of Index Fund Tracking III

Figure: Net asset value returns of five S&P 500 Funds

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Tracking Technology

Tracking error minimization has been widely applied by researchers to solve index tracking problems.

- Meade and Salkin [Meade and Salkin, 1990] employ quadratic programming to construct equity index funds by minimizing tracking errors between index returns and asset returns generated from ARCH process.
- Gilli and Këllezi [Gilli and Këllezi, 2002] use Threshold Accepting to obtain portfolio compositions that track market indexes returns.
- Beasley et al. [Beasley et al., 2003] develop an evolutionary heuristic method to construct a portfolio tracking market returns.
- Maringer [Maringer, 2006, Maringer, 2008] uses Differential Evolution to solve constrained index tracking problems.

Issues in Fund Tracking

The proposed tracking model is based on tracking error minimization; several issues are addressed in this study:

- a multi-period optimization problem (need rebalancing)
- tactical rebalancing strategies
- transaction costs
- cardinality constraints
- objective functions' impact on the tracking performance

Notations

n_{i,T_0}	Number of shares of the i-th equity invested
γ	The transaction costs limiting ratio
P_t	Market value of the tracker at time t
$r_{P,t}$	The tracker return at time t
$r_{I,t}$	Index fund return at time t
$\mathscr{C}_g$	The tracker equity set
w_g^ℓ	Minimum weight of each equity invested
w_g^u	Maximum weight of each equity invested
TC_t	Transaction cost at time t
ρ	Transaction cost coefficient
$Cash_t$	Cash reserve at time t
C	Cash reserve rate
B_t	Sum of the tracker market value and cash reserve at time t
$S_{i,t}$	Per-share market value of the i-th equity at time t
N	Number of available equities in the equity market

The Model: Tracker Construction Stage

The decision variables are different at the construction and rebalancing stage.

Objective Function:

$$\min_{\mathbf{n}} TE = \sqrt{\frac{\sum_t (|r_{P,t} - r_{I,t}|)^2}{T_0 - T_\varpi}} \tag{1}$$

subject to:

$$n_{i,T_0} \in \mathbb{N}^+ \tag{2}$$

$$t \in [T_\varpi, T_0] \tag{3}$$

$$k^{\min} < \sharp\mathscr{C}_g = \sum_{i=1}^{N} \mathbf{I}_{\mathscr{C}_g}(i) \leq k < k^{\max} \tag{4}$$

$$w_g^\ell \leq \frac{n_{i,T_0} \cdot S_{i,T_0}}{P_{T_0}} \leq w_g^u \quad \text{for } i \in \mathscr{C}_g \tag{5}$$

$$TC_{T_0} = \sum_{i \in \mathscr{C}_g} \rho \cdot n_{i,T_0} \cdot S_{i,T_0} \leq \gamma \cdot P_{T_0} \tag{6}$$

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The Model: Tracker Rebalancing Stage

Objective Function:

$$\min_{\delta(\mathbf{n})} TE = \sqrt{\frac{\sum_t (|r_{P,t} - r_{I,t}|)^2}{T_j - T_{j-1}}} \tag{7}$$

subject to

$$\delta(n_{i,T_j}) \in \mathbb{Z} \tag{8}$$

$$n_{i,T_j} \in \mathbb{N}^+ \tag{9}$$

$$t \in [T_{j-1}, T_j] \tag{10}$$

$$k^{\min} < \sharp\mathscr{C}_g = \sum_{i=1}^{N} \mathbf{I}_{\mathscr{C}_g}(i) \leq k < k^{\max} \tag{11}$$

$$w_g^{\ell} \leq \frac{(n_{i,T_{j-1}} + \delta(n_{i,T_j})) \cdot S_{i,T_j}}{P_{T_j}} \leq w_g^{u} \quad \text{for } i \in \mathscr{C}_g \tag{12}$$

$$TC_{T_j} = \sum_{i \in \mathscr{C}_g} 2 \cdot \rho \cdot |n_{i,T_j} - n_{i,T_{j-1}}| \cdot S_{i,T_j} \leq \gamma \cdot P_{T_j} \tag{13}$$

The Model: Calendar Based Rebalancing

This strategy schedules regular rebalancing at a regular calendar interval T_ψ.

1. The model splits the future time horizon $[T_0, T_\omega]$ into M subintervals $[T_0, T_1]$, $[T_1, T_2]$, $\cdots$, $[T_{M-1}, T_\omega]$ according to a fixed calendar interval T_ψ. The interval number M is decided by the length of the rebalancing stage and the time interval: $M = \lfloor (T_\omega - T_0)/T_\psi \rfloor$.
2. At the rebalancing time T_j, the model
   1. decides an optimal set of quantities $\delta(n_{i,T_j})$ based on the market information over the time period $[T_{j-1}, T_j]$,
   2. adjusts portfolio holdings $n_{i,T_j} = n_{i,T_{j-1}} + \delta(n_{i,T_j})$,
   3. updates cash reserves $Cash_{T_j} = Cash_{T_{j-1}} - TC_{T_j}$, and
   4. waits till the next planned rebalancing point $T_{j+1} = T_j + T_\psi$.
3. The model repeats the second step until the end of the rebalancing stage T_ω.

The Model: Tolerance Triggered Rebalancing

1. At each check-point T_j, the tracker has the starting point of the j-th window, $T_{\varsigma,j} = T_j - W_L$ with $j = 1, 2, ..., M$.
2.
   1. If any one of the following conditions is violated: $\sqrt{\frac{1}{W_L} \cdot \sum_{t=T_{\varsigma,j}}^{T_j} |r_{P,t} - r_{I,t}|^2} < \xi_1$, $\frac{n_{i,T_j} \cdot S_{i,T_j}}{P_{T_j}} > x_g^{\ell}$, and $\frac{n_{i,T_j} \cdot S_{i,T_j}}{P_{T_j}} < x_g^{u}$, the model
      (i) finds an optimal set of $\delta(n_{i,T_j})$ based on the market information in the time period $[T_{\varsigma,j}, T_j]$,
      (ii) adjusts portfolio holdings: $n_{i,T_j} = n_{i,T_{j-1}} + \delta(n_{i,T_j})$,
      (iii) updates cash reserves: $Cash_{T_j} = Cash_{T_{j-1}} - TC_{T_j}$;
   2. otherwise the model keeps the holdings unchanged: $n_{i,T_j} = n_{i,T_{j-1}}$;
   3. the model waits till the next check-point $T_{j+1} = T_j + \wp$.
3. The model repeats the second step up till the end of rebalancing stage T_ω.

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The Model: Two Extensions of TE Optimization

In addition to the classic TE optimization, we consider two extensions from index tracking.

- Extension to Include Excess Return ([Gilli and Këllezi, 2002])

$$ER = \frac{1}{T_N} \sum_t (r_{P,t} - r_{I,t}) \tag{14}$$

$$\min \;\; \lambda \cdot TE - (1 - \lambda) \cdot ER \tag{15}$$

- Extension to Include Loss Aversion ([Maringer, 2008])

$$\widetilde{\Delta}_r = \begin{cases} r_{P,t} - r_{I,t} & r_{P,t} \geq r_{I,t} \\ (r_{P,t} - r_{I,t}) \cdot \vartheta & r_{P,t} < r_{I,t} \end{cases} \tag{16}$$

$$\min \;\; \widetilde{TE} = \sqrt{\frac{1}{T_N} \sum_t (\widetilde{\Delta}_r)^2} \tag{17}$$

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The Model: The Optimization Method

Algorithm 2.1: DIFFERENTIAL EVOLUTION: FITNESS(v_p)

randomly initialize population of vectors v_p, p= 1...P;
while do
  for all current solutions v_p, p=1...P **do**
    randomly pick $p_1 \neq p_2 \neq p_3 \neq p$;
    $v_c[i] \leftarrow v_{p1}[i] + (F + z_1[i])(v_{p2}[i] - v_{p3}[i] + z_2[i])$ at probability π_1;
    or $v_c[i] \leftarrow v_p[i]$ at probability $1 - \pi_1$;
  interpret v_c into equity weights;
  Compute the Fitness of v_c;
  end for;
  for the current solution v_p, $p = 1...P$ **do**
    if Fitness(v_c) > Fitness(v_p) **then**;
      $v_p \leftarrow v_c$;
    end if;
  end for;
until halting criterion met;

Figure: Differential Evolution

Data

- Five S&P 500 index funds (the trackers are identified by different colours in the following: blue, green, red, cyan and magenta)
  1. ETRADE S&P 500 Index (ETSPX, 0.09%)
  2. VANGUARD 500 Index (VFINX, 0.15%)
  3. UBS S&P 500 Index A (PSPIX, 0.70%)
  4. USAA S&P 500 Index (USSPX, 0.19%)
  5. TIAA-CREF S&P 500 Index Retire (TRSPX, 0.31%)
- A total of 445 equites were used to track the five index funds.
- The equities have price sequences with 1,043 observations (January 2004 to December 2007).
- The first 250 observations were used to construct trackers, which would be held from the beginning of 2005. The latest 60 observations at each rebalancing point were employed to decide on the optimal adjusted quantities for rebalancing.

Settings I

Parameter setting: Initial budget $B_t = 10,000,000$ dollars; Cash Reserve Rate $C = 10\%$; Transaction Cost Limiting Rate $\gamma = 1\%$; Transaction Cost Coefficient $\rho = 0.1\%$; Cardinality Size $k = 5$.

- Calendar based rebalancing: Rebalancing Interval $T_\psi = 60$
- Tolerance triggered rebalancing: Window Size $W_L = 60$; Step Size $\wp = 10$; TE Tolerance $\xi_1 = 0.004$; Weights Tolerance $1\% < x_{i,t} < 50\%$
- TE with ER Optimization: Weighted Difference $\lambda = 1/2$
- TE under loss aversion: Aversion Coefficient $\vartheta = 2$

Technical parameters of DE algorithm: Population size and iteration number were set at 1,000 and 4,000; the factor F was set at a value 0.5; and the cross-over probability π_1 was at 60%. The parameters were used to generate the artificial noise: $\pi_2 = 50\%$, $\pi_3 = 10\%$, $\sigma_1^2 = 0.1$ and $\sigma_2^2 = 0.1$.

In-Sample First and Second Moments of Funds and Trackers: the Calendar Rebalancing I

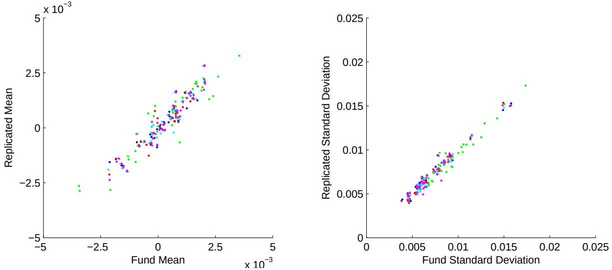

Figure: Tracking Error Optimization

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In-Sample First and Second Moments of Funds and Trackers: the Calendar Rebalancing II

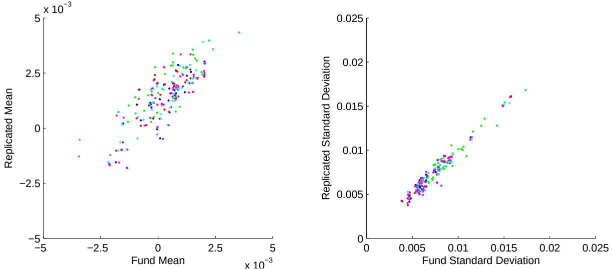

Figure: Tracking Error with Excess Return Optimization

In-Sample First and Second Moments of Funds and Trackers: the Calendar Rebalancing III

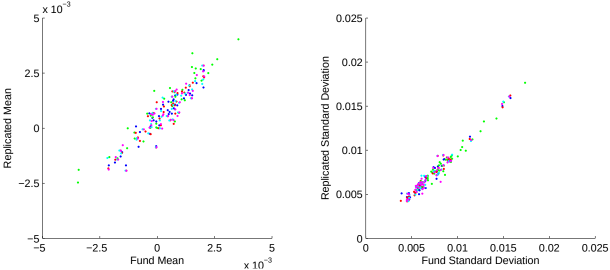

Figure: Tracking Error with Loss Aversion Optimization

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In-Sample Monthly Tracking Error and Excess Sharpe Ratio: the Calendar Rebalancing I

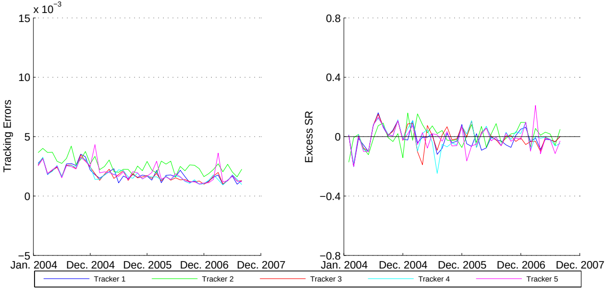

Figure: Tracking Error Optimization

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In-Sample Monthly Tracking Error and Excess Sharpe Ratio: the Calendar Rebalancing II

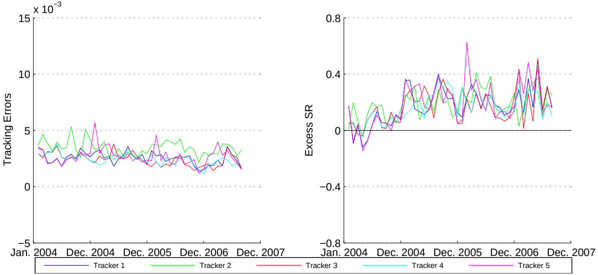

Figure: Tracking Error with Excess Return Optimization

In-Sample Monthly Tracking Error and Excess Sharpe Ratio: the Calendar Rebalancing III

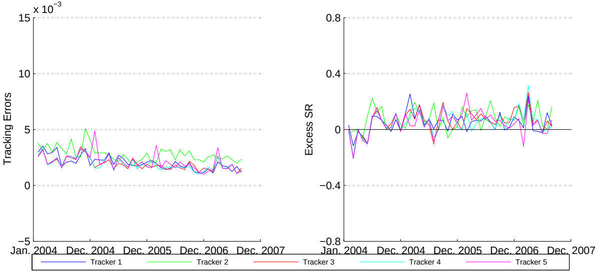

Figure: Tracking Error with Loss Aversion Optimization

Out-of-Sample First and Second Moments of Funds and Trackers: the Calendar Based Rebalancing I

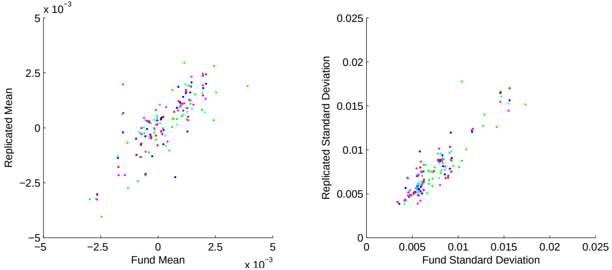

Figure: Tracking Error Optimization

Out-of-Sample First and Second Moments of Funds and Trackers: the Calendar Based Rebalancing II

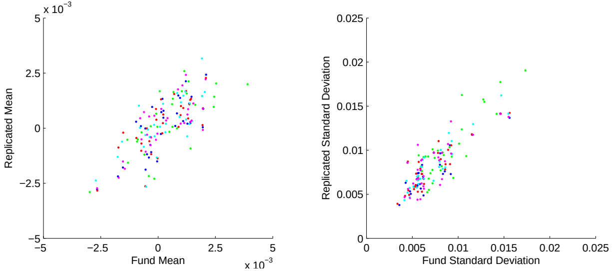

Figure: Tracking Error with Excess Return Optimization

Out-of-Sample First and Second Moments of Funds and Trackers: the Calendar Based Rebalancing III

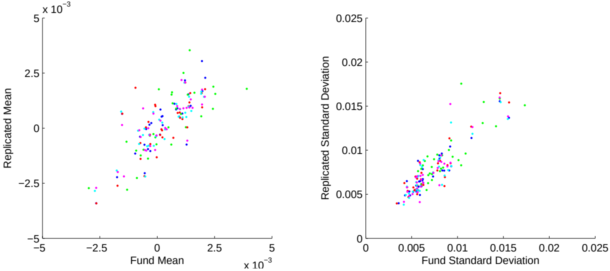

Figure: Tracking Error with Loss Aversion Optimization

Out-of-Sample First and Second Moments of Funds and Trackers: the Tolerance Triggered Rebalancing I

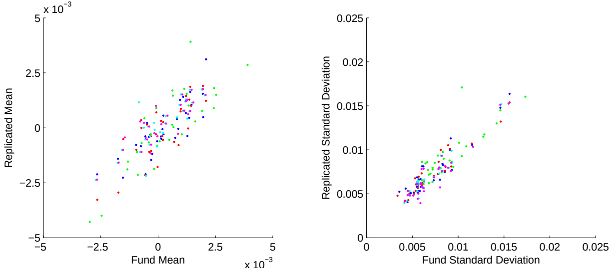

Figure: Tracking Error Optimization

Out-of-Sample First and Second Moments of Funds and Trackers: the Tolerance Triggered Rebalancing II

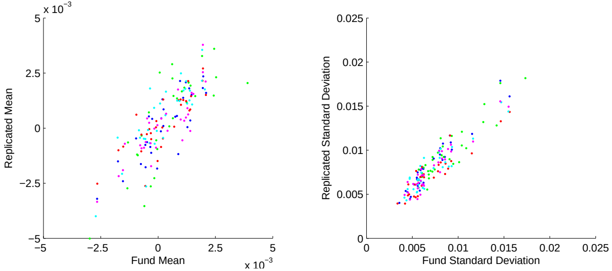

Figure: Tracking Error with Excess Return Optimization

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Out-of-Sample First and Second Moments of Funds and Trackers: the Tolerance Triggered Rebalancing III

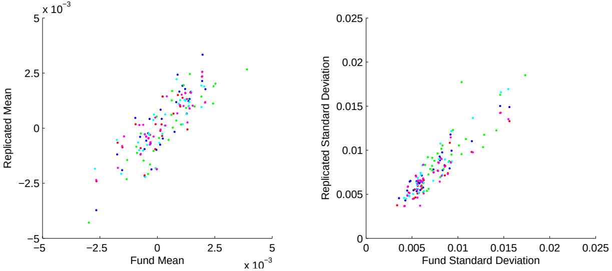

Figure: Tracking Error with Loss Aversion Optimization

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Regression Analysis of Actual and Replicated Return I

Table: Return Mean Analysis

	TE Opt.		Ext. 1		Ext. 2	
	C.	T.	C.	T.	C.	T.
SSE.(10^{-3})	0.0983	0.0818	0.1296	0.1427	0.1110	0.0919
R^2	0.6494	0.6924	0.5576	0.6682	0.6117	0.7094
$\bar{\alpha}$ (10^{-3})	-0.0199	-0.1942	-0.1054	-0.0692	-0.0969	-0.1239
std.(α) (10^{-3})	0.0661	0.0603	0.0758	0.0796	0.0702	0.0639
$p(\alpha \neq 0)$	0.7436	0.0016	0.1665	0.3860	0.1696	0.0542
$\bar{\beta}$	0.8950	0.8999	0.8474	1.1238	0.8771	0.9931
std.(β)	0.0532	0.0485	0.0610	0.0640	0.0565	0.0514
$p(\beta \neq 1)$	0.0497	0.0413	0.0124	0.0536	0.0308	0.8966

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Regression Analysis of Actual and Replicated Return II

Table: Return Standard Deviation Analysis

	TE Opt.		Ext. 1		Ext. 2	
	C.	T.	C.	T.	C.	T.
SSE(10^{-3})	0.2516	0.1623	0.3170	0.1736	0.2793	0.2316
R^2	0.8206	0.8650	0.7597	0.8664	0.7908	0.8294
$\bar{\alpha}(10^{-3})$	0.5485	0.7363	1.6700	1.1030	1.0100	0.6546
std.$(\alpha)(10^{-3})$	0.2920	0.2346	0.3278	0.2426	0.3077	0.2802
$p(\alpha \neq 0)$	0.0623	0.0020	$< .0001$	$< .0001$	0.0013	0.0208
$\bar{\beta}$	0.9654	0.9176	0.9008	0.9548	0.9245	0.9548
std.(β)	0.0365	0.0293	0.0410	0.0303	0.0384	0.0350
$p(\beta \neq 1)$	0.3472	0.0051	0.0163	0.1362	0.0495	0.1973

Out-of-Sample Tracking Errors (left – calendar based rebalancing, right – tolerance triggered rebalancing) I

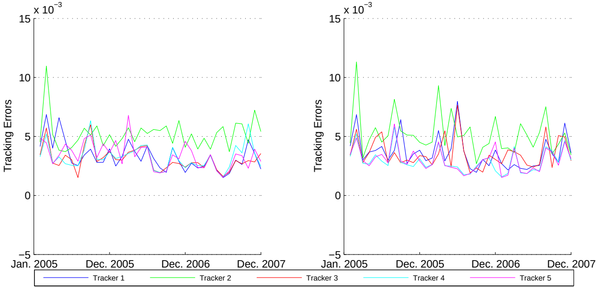

Figure: Tracking Error Optimization

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Out-of-Sample Tracking Errors (left – calendar based rebalancing, right – tolerance triggered rebalancing) II

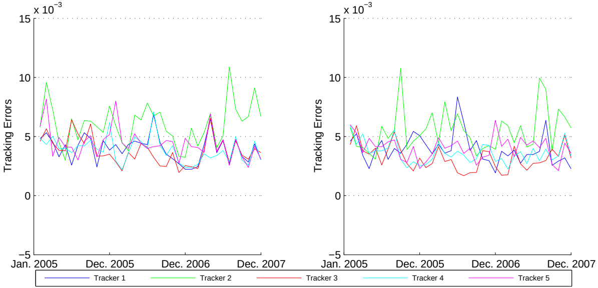

Figure: Tracking Error with Excess Return Optimization

Out-of-Sample Tracking Errors (left – calendar based rebalancing, right – tolerance triggered rebalancing) III

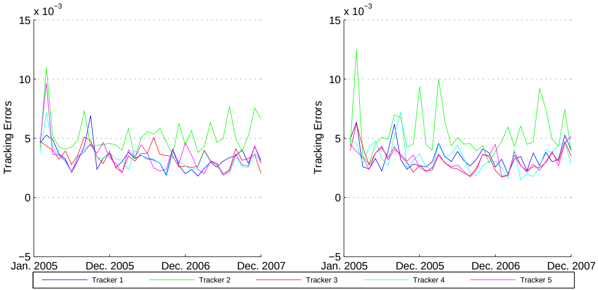

Figure: Tracking Error with Loss Aversion Optimization

Out-of-Sample Excess Sharpe Ratios (left – calendar based rebalancing, right – tolerance triggered rebalancing) I

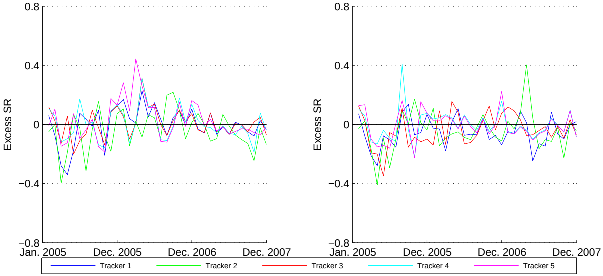

Figure: Tracking Error Optimization

Out-of-Sample Excess Sharpe Ratios (left – calendar based rebalancing, right – tolerance triggered rebalancing) II

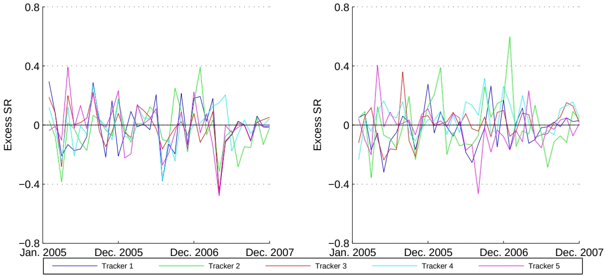

Figure: Tracking Error with Excess Return Optimization

Out-of-Sample Excess Sharpe Ratios (left – calendar based rebalancing, right – tolerance triggered rebalancing) III

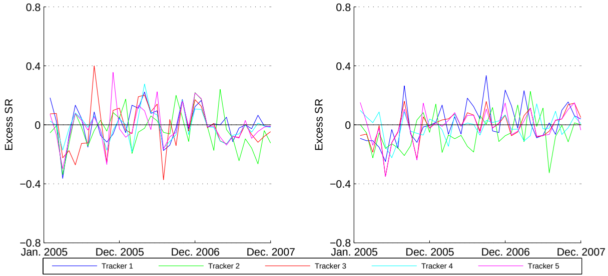

Figure: Tracking Error with Loss Aversion Optimization

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Number of Rebalances at the Rebalancing Stage

Table: Number of Rebalances

		Tracker 1	Tracker 2	Tracker 3	Tracker 4	Tracker 5
TE Opt.	2005	3	7	3	2	4
	2006	3	8	2	0	0
	2007	2	7	3	0	0
Ext. 1	2005	3	9	4	5	7
	2006	5	13	0	0	2
	2007	2	13	1	1	6
Ext. 2	2005	3	8	3	4	4
	2006	0	7	0	1	0
	2007	1	9	1	2	1

–Return Tables–

Conclusions

- The regression test results support that the model replicates the first two moments of index fund returns by using only five equities.
- The tolerance triggered rebalancing outperformed the calendar based rebalancing in terms of both tracking ability and cost-efficiency.
- The study of the different sample size impact on the tracker performance revealed that using a large data sample size might result in high tracking errors and losses.
- The tracking error optimization under loss aversion with tolerance triggered rebalancing dominates other combinations studied in this work.

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Thanks


I have no idea what you're talking about...
PYZAM.com
...so here's a bunny with a pancake on its head.