Optimising Risk and Reward of Financial Portfolios with Threshold Accepting

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introduction

portfolio optimisation: allocate wealth among $n_{\mathcal{A}}$ assets

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$$w'\mu - \frac{\gamma}{2} w'\Sigma w$$

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1: collect historical data $R_{\;(T \times n_{\mathcal{A}})}$
2: estimate $\hat{\mu} = \frac{1}{T}\iota' R$
3: estimate $\hat{\Sigma} = \frac{1}{T} R' \big(I - \frac{1}{T}\iota\iota'\big) R$

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estimation problems: Jobson and Korkie (1980), Jorion (1985), Jorion (1986), Best and Grauer (1991), Chopra et al. (1993), Board and Sutcliffe (1994) and many others

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theoretical concerns: Artzner et al. (1999), Pedersen and Satchell (1998), Pedersen and Satchell (2002)

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aim of research:

- test alternative risk measures & objective functions empirically
- test alternative estimation and scenario generation methods

outline

- alternative objective functions

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- data

outline

- alternative objective functions
- data
- optimisation

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- alternative objective functions
- data
- optimisation
- empirical results

alternative objective functions: building blocks

$$\underbrace{w'\mu}_{\text{reward}} - \underbrace{\frac{\gamma}{2}w'\Sigma w}_{\text{risk}}$$

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replace reward and risk by alternative functions

$$\min_{w} \frac{\text{risk}}{\text{reward}}$$

alternative objective functions: building blocks

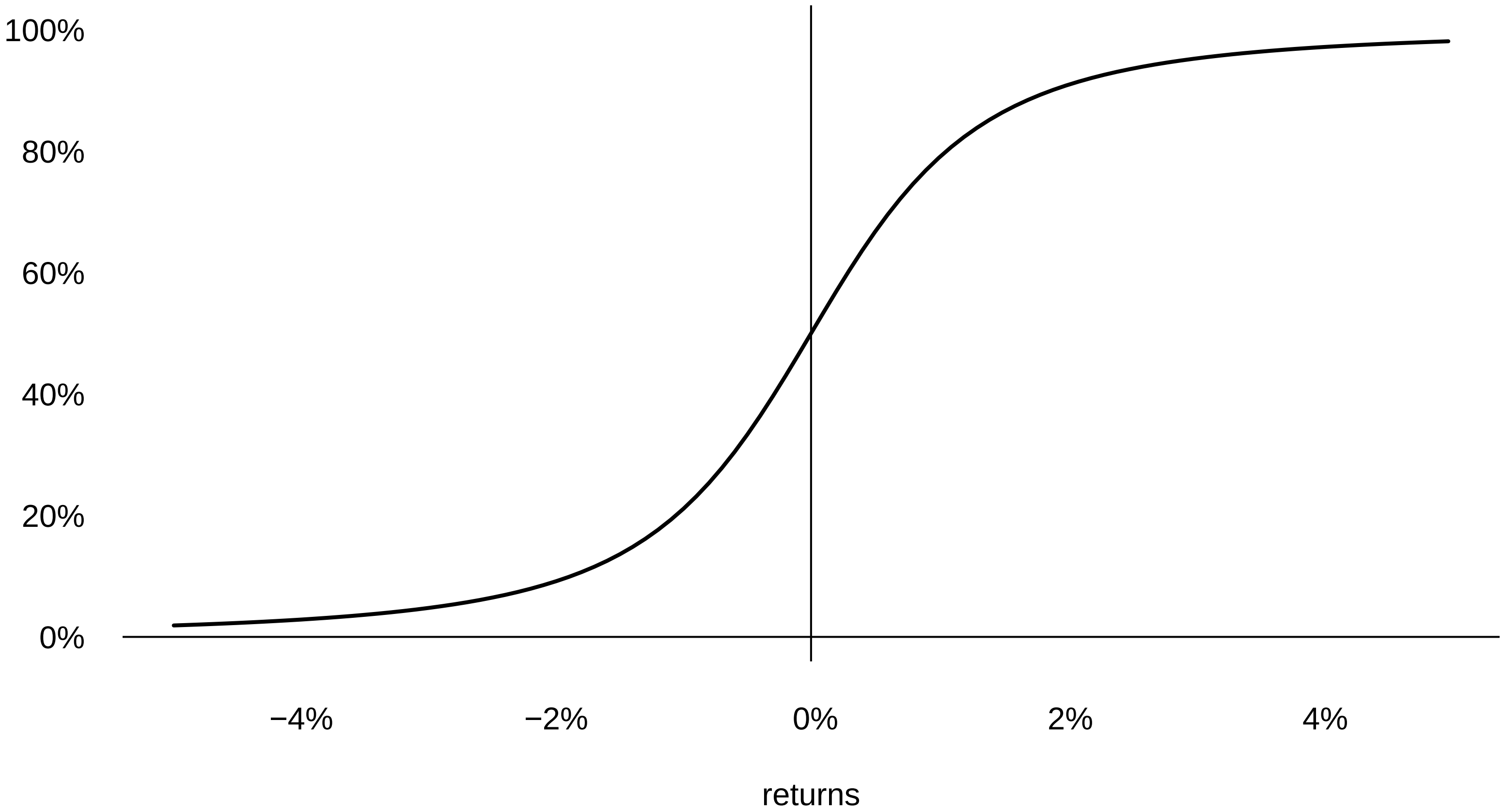
$$\underbrace{w'\mu}_{\text{reward}} - \underbrace{\frac{\gamma}{2}w'\Sigma w}_{\text{risk}}$$

replace reward and risk by alternative functions

$$\min_w \frac{\text{risk}}{\text{reward}} = \Phi$$

alternative objective functions: building blocks

based on distribution of portfolio returns



alternative objective functions: building blocks

based on distribution of portfolio returns

- moments (variance, skewness, . . .)
- conditional moments (expected shortfall, . . .), partial moments (semivariance, . . .)
- quantiles (VaR, . . .), corresponding probabilities (shortfall probability, . . .)

alternative objective functions: building blocks

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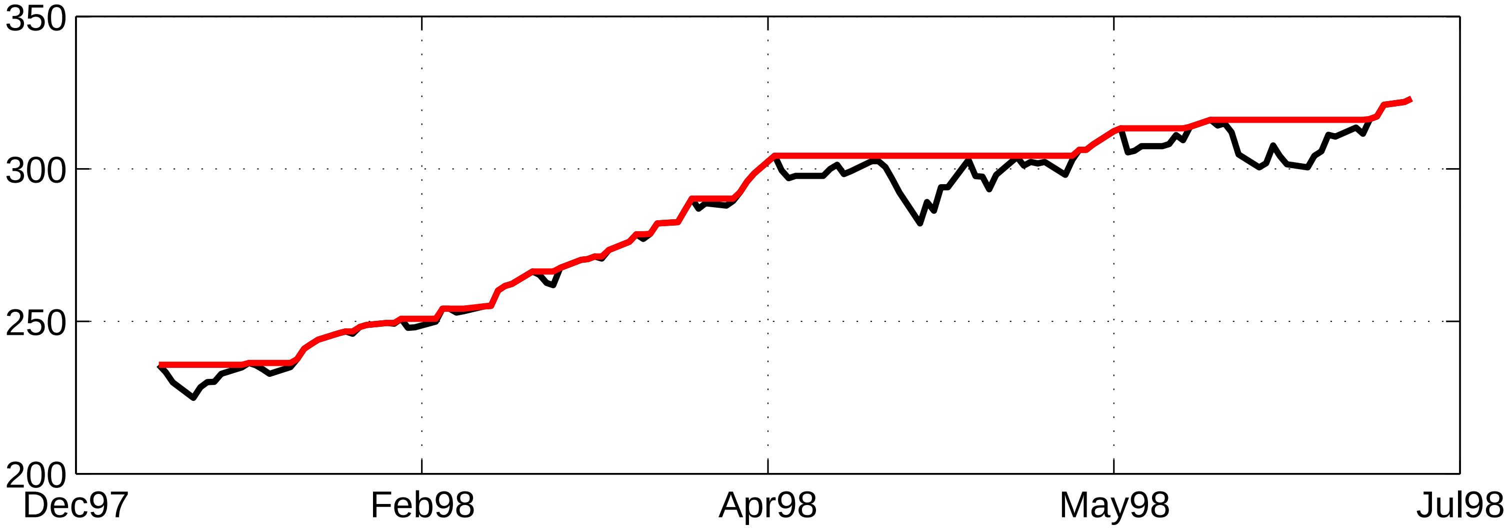
based on trajectory of portfolio wealth

- drawdown ($\mathcal{D}$), time under water, ...

alternative objective functions: building blocks



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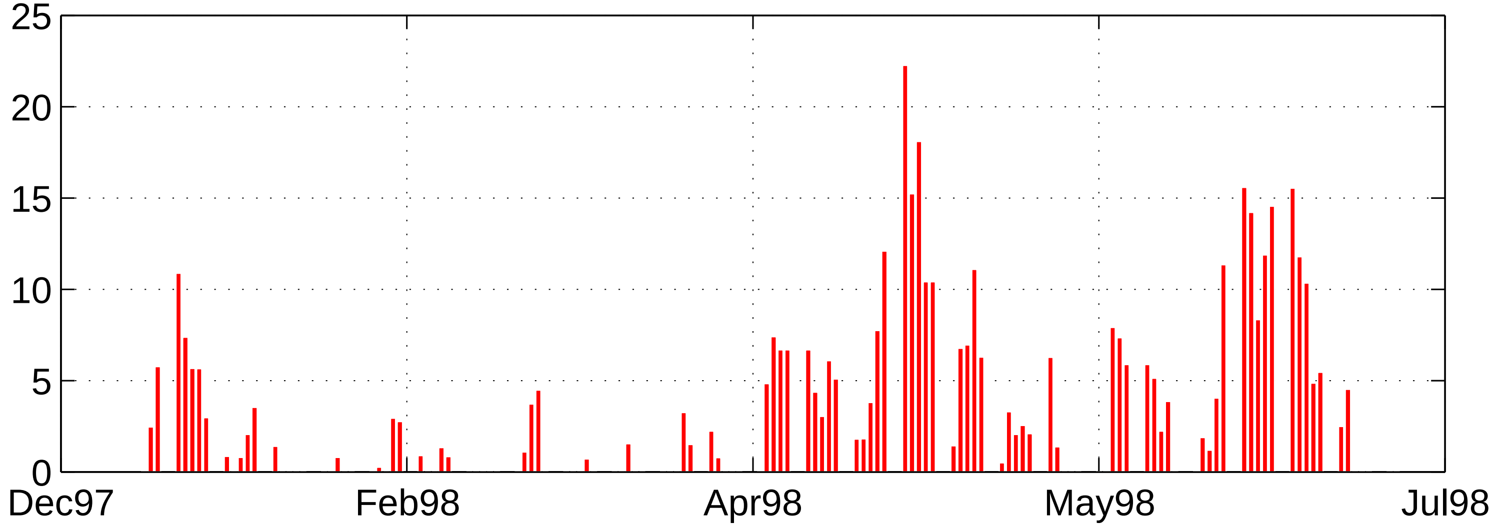

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objective function: do as you please

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based on trajectory of portfolio wealth

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objective function: do as you please

alternative objective functions: partial moments

capture non-symmetrical returns Bawa (1975); Fishburn (1977):

$$r = \underbrace{r_d}_{\text{desired return}} + \underbrace{(r - r_d)^+}_{\text{upside}} - \underbrace{(r_d - r)^+}_{\text{downside}}$$

alternative objective functions: partial moments

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$$\mathcal{P}_\gamma^+(r_d) = \frac{1}{T} \sum_{r > r_d} (r - r_d)^\gamma \; ,$$

$$\mathcal{P}_\gamma^-(r_d) = \frac{1}{T} \sum_{r < r_d} (r_d - r)^\gamma \; .$$

example: semi-variance

alternative objective functions: conditional moments

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example: Expected Shortfall

alternative objective functions: conditional moments

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$$r = \underbrace{r_d}_{\text{desired return}} + \underbrace{(r - r_d)^+}_{\text{upside}} - \underbrace{(r_d - r)^+}_{\text{downside}}$$

conditional vs partial moments

$$\mathcal{P}_\gamma^+(r_d) = \mathcal{C}_\gamma^+(r_d)\,\underbrace{\mathcal{P}_0^+(r_d)}_{\pi \text{ of } r>r_d}$$

$$\mathcal{P}_\gamma^-(r_d) = \mathcal{C}_\gamma^-(r_d)\,\underbrace{\mathcal{P}_0^-(r_d)}_{\pi \text{ of } r<r_d}$$

alternative objective functions: quantiles

$$\mathcal{Q}_q = \mathrm{CDF}^{-1}(q) = \min\{r \mid \mathrm{CDF}(r) \geq q\}\,,$$

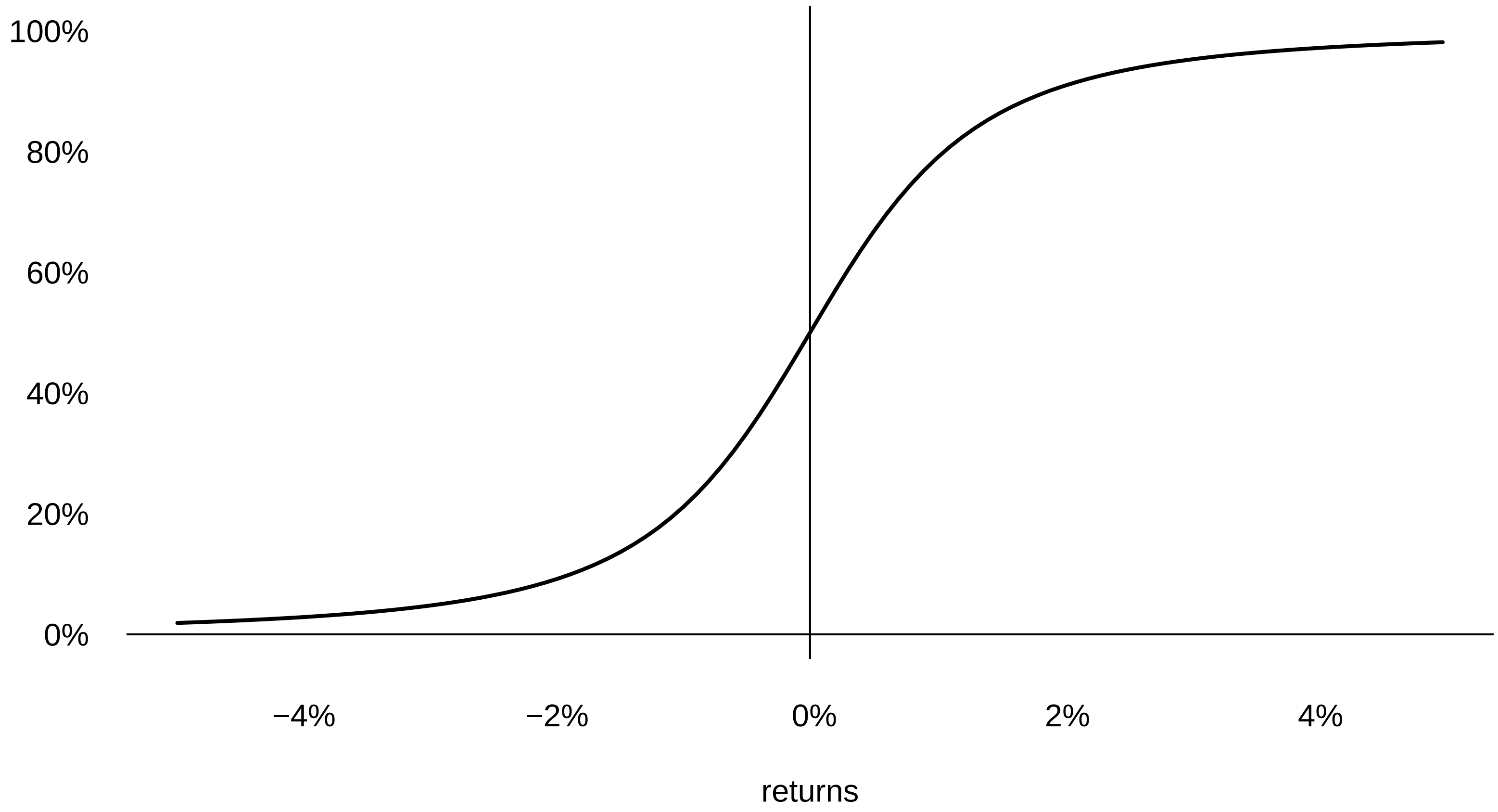
example: VaR

alternative objective functions: examples

reward	risk	
constant	$\mathcal{C}_1^-(\mathcal{Q}_q)$	minimise Expected Shortfall for qth quantile
constant	$\mathcal{Q}_0$	minimise maximum loss
$\frac{1}{n_\mathcal{S}}\sum r$	$\sqrt{\mathcal{P}_2^-(r_d)}$	Sortino ratio
$\mathcal{P}_1^+(r_d)$	$\sqrt{\mathcal{P}_2^-(r_d)}$	Upside Potential ratio
$\mathcal{P}_1^+(r_d)$	$\mathcal{P}_1^-(r_d)$	Omega for threshold r_d
$\frac{1}{n_\mathcal{S}}\sum r$	$\mathcal{D}_{\max}$	Calmar ratio
$\mathcal{C}_\gamma^+(\mathcal{Q}_p)$	$\mathcal{C}_\delta^-(\mathcal{Q}_q)$	Rachev Generalised ratio for exponents γ and δ

estimation

estimation



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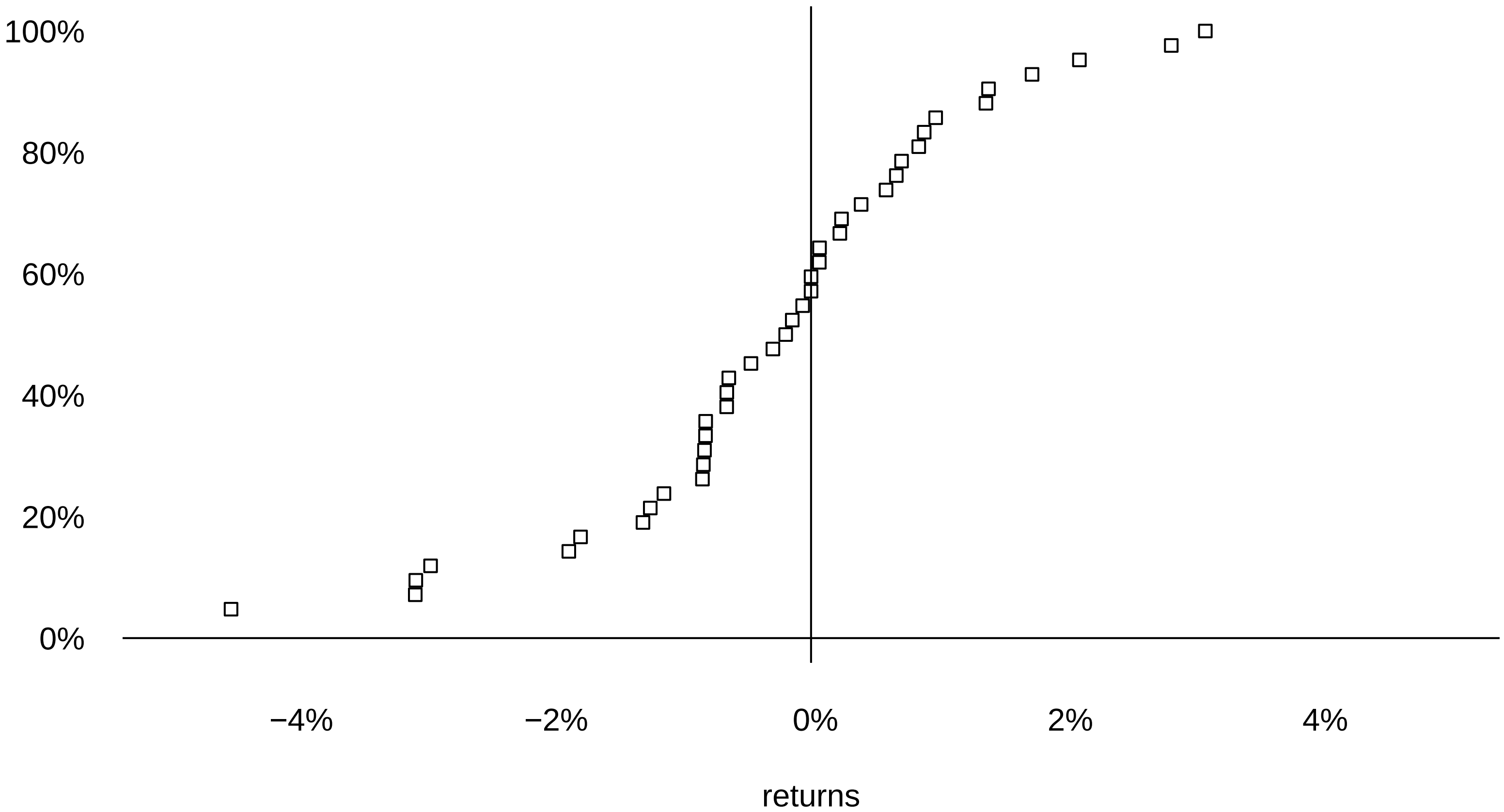

empirical distribution of portfolio returns
(order statistics $r_{[1]} \leq r_{[2]} \leq \cdots \leq r_{[T]}$)

estimation: scenario generation

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- historical returns

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- historical returns
- bootstrapping: single observations, blocks . . .

estimation: scenario generation

- historical returns
- bootstrapping: single observations, blocks ...
- bootstrapping from model residuals

estimation

bootstrapping returns (r^B) from a simple regression model:

$$r_{it} = \alpha_i + \beta_i \, r_{Mt} + \cdots + \epsilon_{it} \qquad \begin{array}{l} i = 1, \ldots, n_{\mathcal{A}} \\ t = 1, \ldots, T \end{array}$$

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regressors: indices, PCA . . .

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regressors: indices, PCA ...

estimate $\hat{\alpha}_i$, $\hat{\beta}_i$, $\ldots i = 1, \ldots, n_{\mathcal{A}}$ from model
for $k = 1 : n_S$ **do**
  draw with replacement $\tau_M \in \{1, \ldots, T\}$
  for $i = 1 : n_{\mathcal{A}}$ **do**
    draw with replacement $\tau_i \in \{1, \ldots, T\}$
    $r^B_{ik} = \hat{\alpha}_i + \hat{\beta}_i \, r_{M\tau_M} + \epsilon_{i\tau_i}$
  end for
end for

estimation

estimation

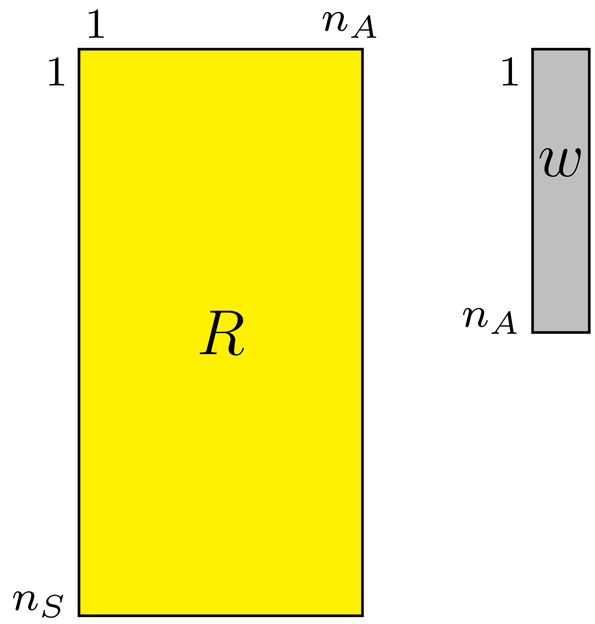
arbitrage opportunities in data sample

$$\min_w w'\iota \qquad \max_w (Rw)'\iota$$

$$Rw \geq 0 \qquad Rw \geq 0$$

$$w'\iota = 0$$

(see for example Scherer (2004))



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$$\min_{x} \ \Phi(x)$$

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$$x_j^{\text{inf}} \leq x_j \leq x_j^{\text{sup}} \qquad j \in \mathcal{J}$$

$$K_{\text{inf}} \leq \#\{\mathcal{J}\} \leq K_{\text{sup}}$$

$$\vdots$$

(x = numbers of shares, $\mathcal{A}$ = all assets, $\mathcal{J}$ = assets included in portfolio)

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Threshold Accepting: Dueck and Scheuer (1990), Winker (2001),
Matlab code available from `http://comisef.eu`

optimisation

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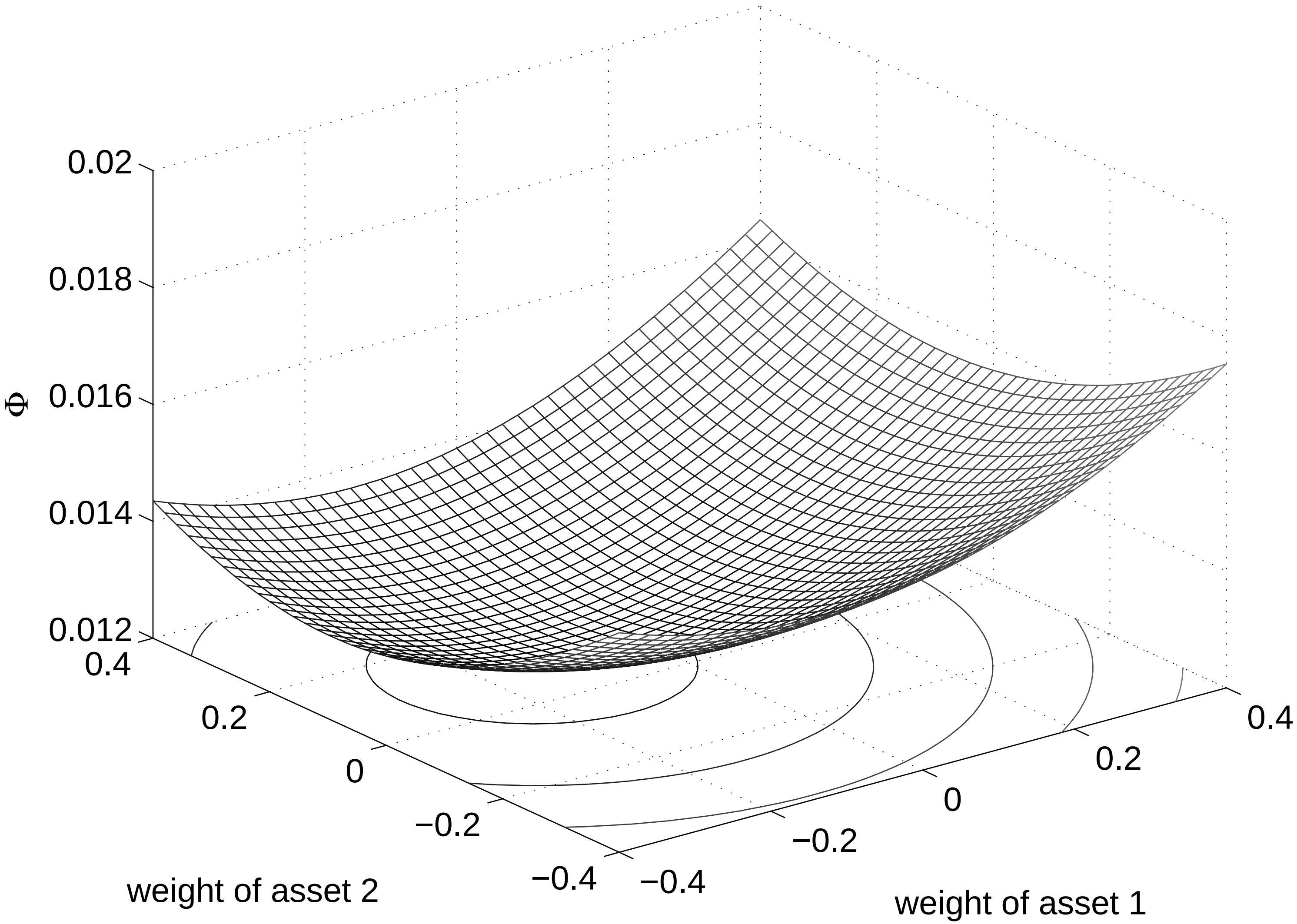

optimisation: threshold accepting

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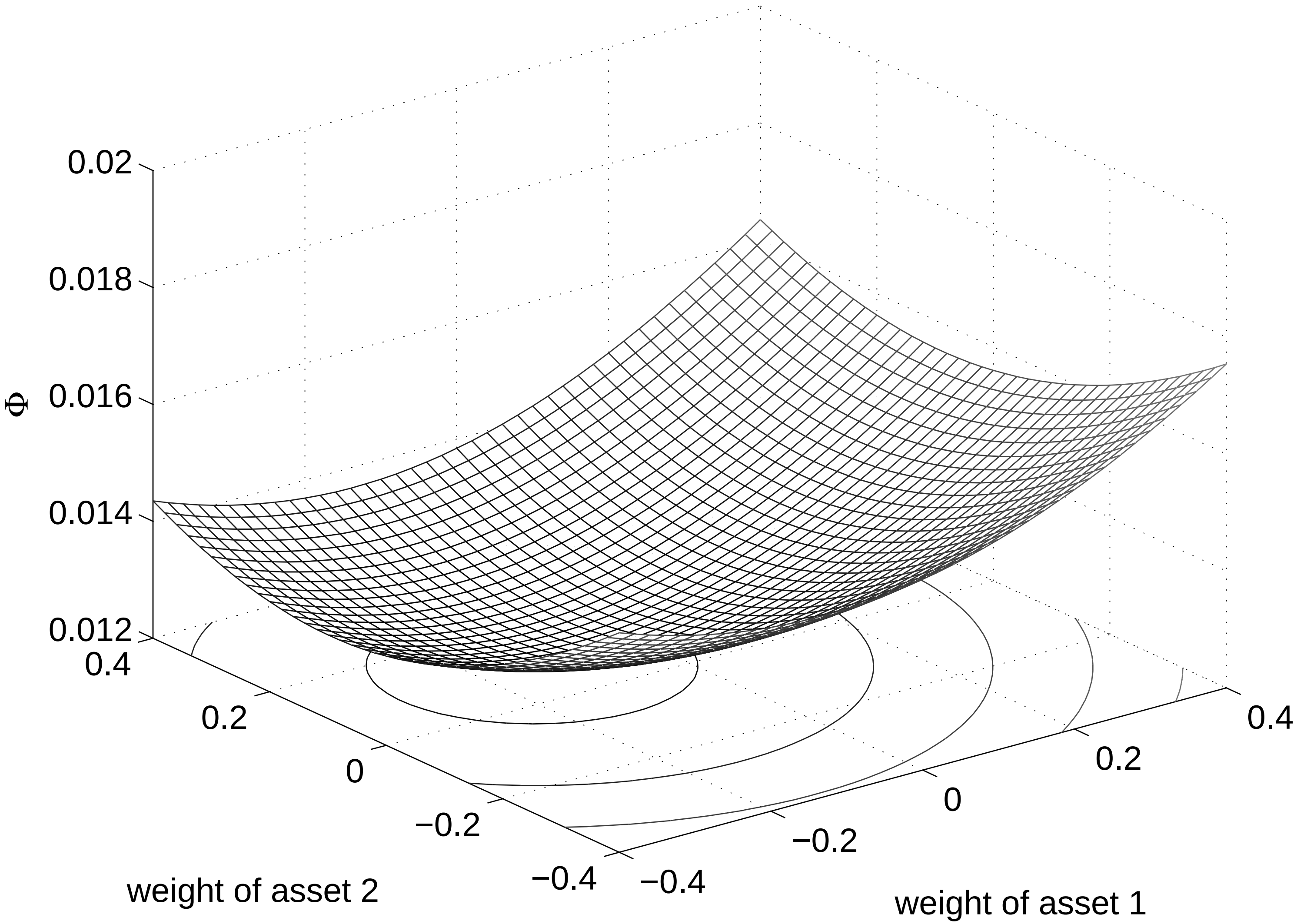
initialise n_{Steps}

randomly generate current solution $x^{\text{c}} \in \mathcal{X}$

for $i = 1 : n_{\text{Steps}}$
    generate $x^{\text{n}} \in \mathcal{N}(x^{\text{c}})$ and compute $\Delta = \Phi(x^{\text{n}}) - \Phi(x^{\text{c}})$
    if $\Delta < 0$ **then** $x^{\text{c}} = x^{\text{n}}$
end for

$x^{\text{sol}} = x^{\text{c}}$

optimisation: threshold accepting



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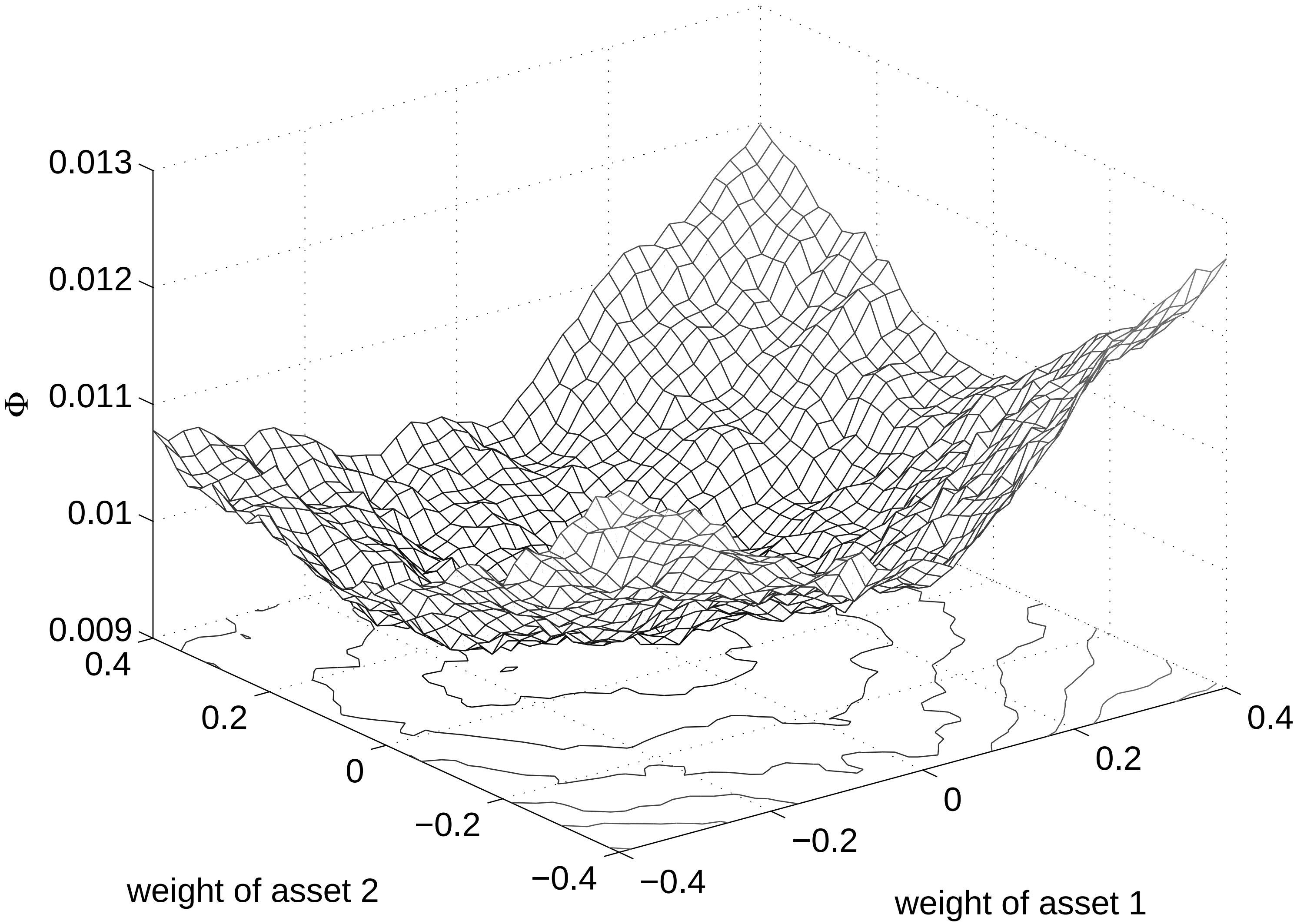

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initialise n_{Steps}

randomly generate current solution $x^{\text{c}} \in \mathcal{X}$

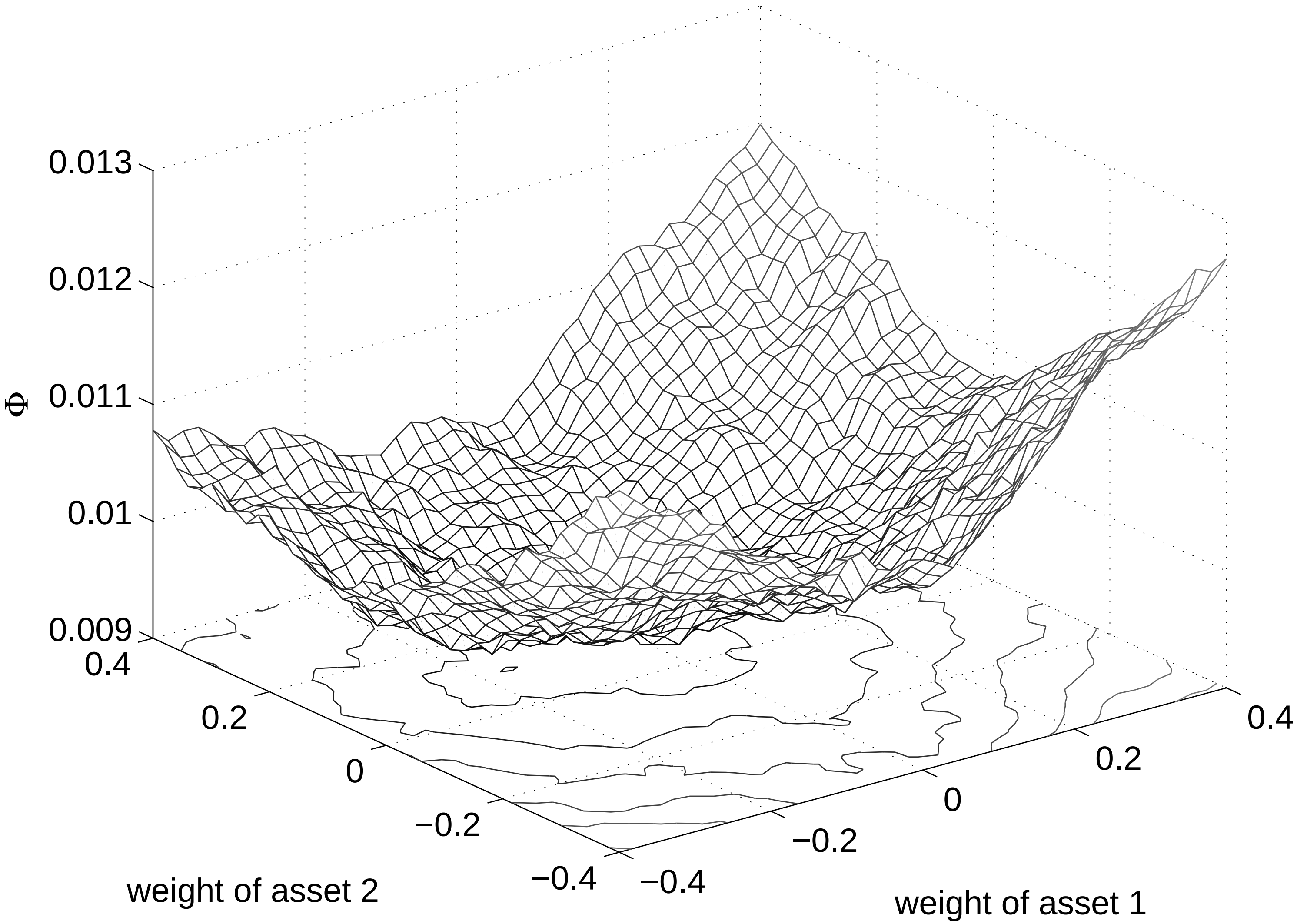
for $i = 1 : n_{\text{Steps}}$
    generate $x^{\text{n}} \in \mathcal{N}(x^{\text{c}})$ and compute $\Delta = \Phi(x^{\text{n}}) - \Phi(x^{\text{c}})$
    if $\Delta < \tau$ **then** $x^{\text{c}} = x^{\text{n}}$
end for

$x^{\text{sol}} = x^{\text{c}}$

optimisation: threshold accepting

initialise n_{Steps} and n_{Rounds}
compute threshold sequence τ
randomly generate current solution $x^{\text{c}} \in \mathcal{X}$
for $r = 1 : n_{\text{Rounds}}$
    for $i = 1 : n_{\text{Steps}}$
        generate $x^{\text{n}} \in \mathcal{N}(x^{\text{c}})$ and compute $\Delta = \Phi(x^{\text{n}}) - \Phi(x^{\text{c}})$
        if $\Delta < \tau_r$ **then** $x^{\text{c}} = x^{\text{n}}$
    end for
end for
$x^{\text{sol}} = x^{\text{c}}$

optimisation: threshold accepting



optimisation: results

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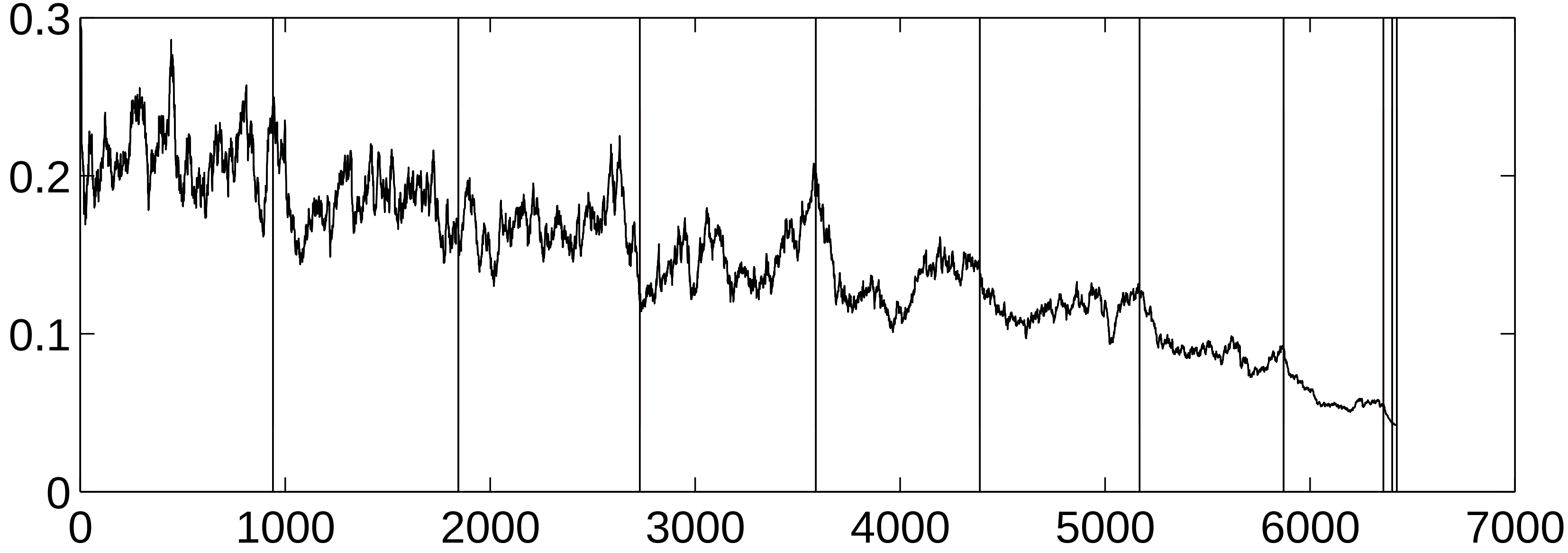

optimisation: constraints

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- discard infeasible solutions

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- always construct feasible solutions
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optimisation: constraints

- discard infeasible solutions
- always construct feasible solutions
    - example: budget constraint
- repair solutions
    - example: sector constraints
- penalise infeasible solutions

why a heuristic?

why a heuristic?

from Zanakis and Evans (1981, p. 85)

"[. . .]
WHY AND WHEN TO USE HEURISTICS

There are several instances where the use of heuristics is desirable and advantageous:

(1) Inexact or limited data used to estimate model parameters may inherently contain errors much larger than the "suboptimality" of a good heuristic
[. . .]"

data and methodology

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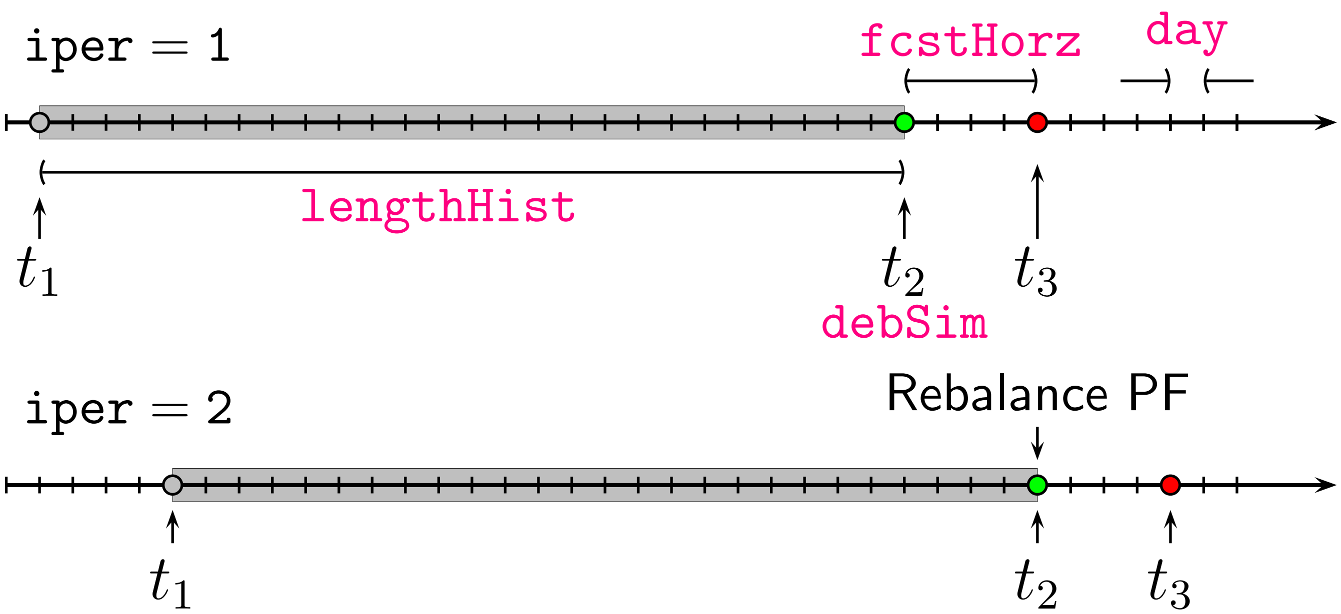

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$$
\begin{aligned}
&\min_{w} \; w' \, \hat{\Sigma} \, w \\
&\textstyle\sum_{j \in \mathcal{J}} w_j = 1 \\
&0 \le w_j \le w_j^{\text{sup}} \qquad j = 1, \ldots, n_{\mathcal{A}}
\end{aligned}
$$

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- optimisation with maximum holding size and sector allocation constraints done with Matlab's `quadprog`.

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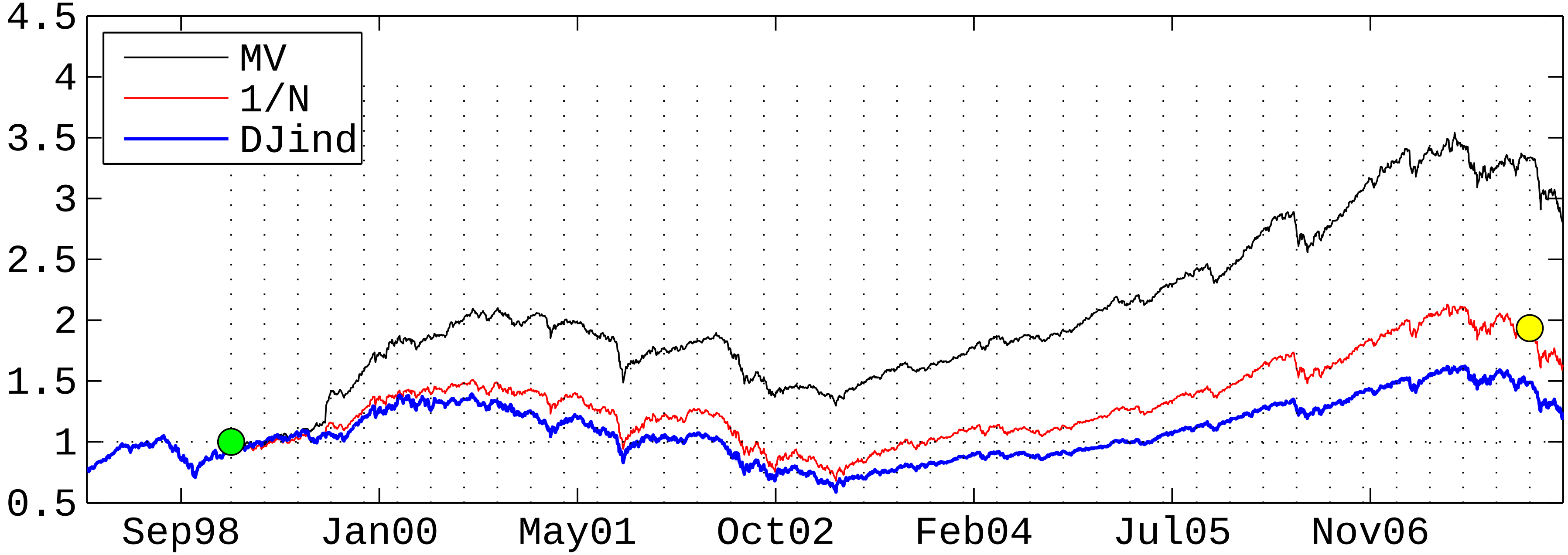

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introducing uncertainty:

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introducing uncertainty: draw T daily data with replacement, compute minimum-variance portfolio from bootstrapped time series

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introducing uncertainty: draw T daily data with replacement, compute minimum-variance portfolio from bootstrapped time series

for $k = 1 : 100$ **do**
    for $j = 1 : T$ **do**
        draw with replacement $\tau \in \{1, \ldots, T\}$
        $R^B_{j\bullet} = R_{\tau\bullet}$
    end for
    compute MV portfolio
end for

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introducing uncertainty: draw T daily data with replacement, compute minimum-variance portfolio from bootstrapped time series

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2:     **for** $j = 1 : T$ **do**
3:         draw with replacement $\tau \in \{1, \ldots, T\}$
4:         $R^B_{j\bullet} = R_{\tau\bullet}$
5:     **end for**
6:     compute MV portfolio
7: **end for**

alternatively: use jackknife

benchmark: MV long-only

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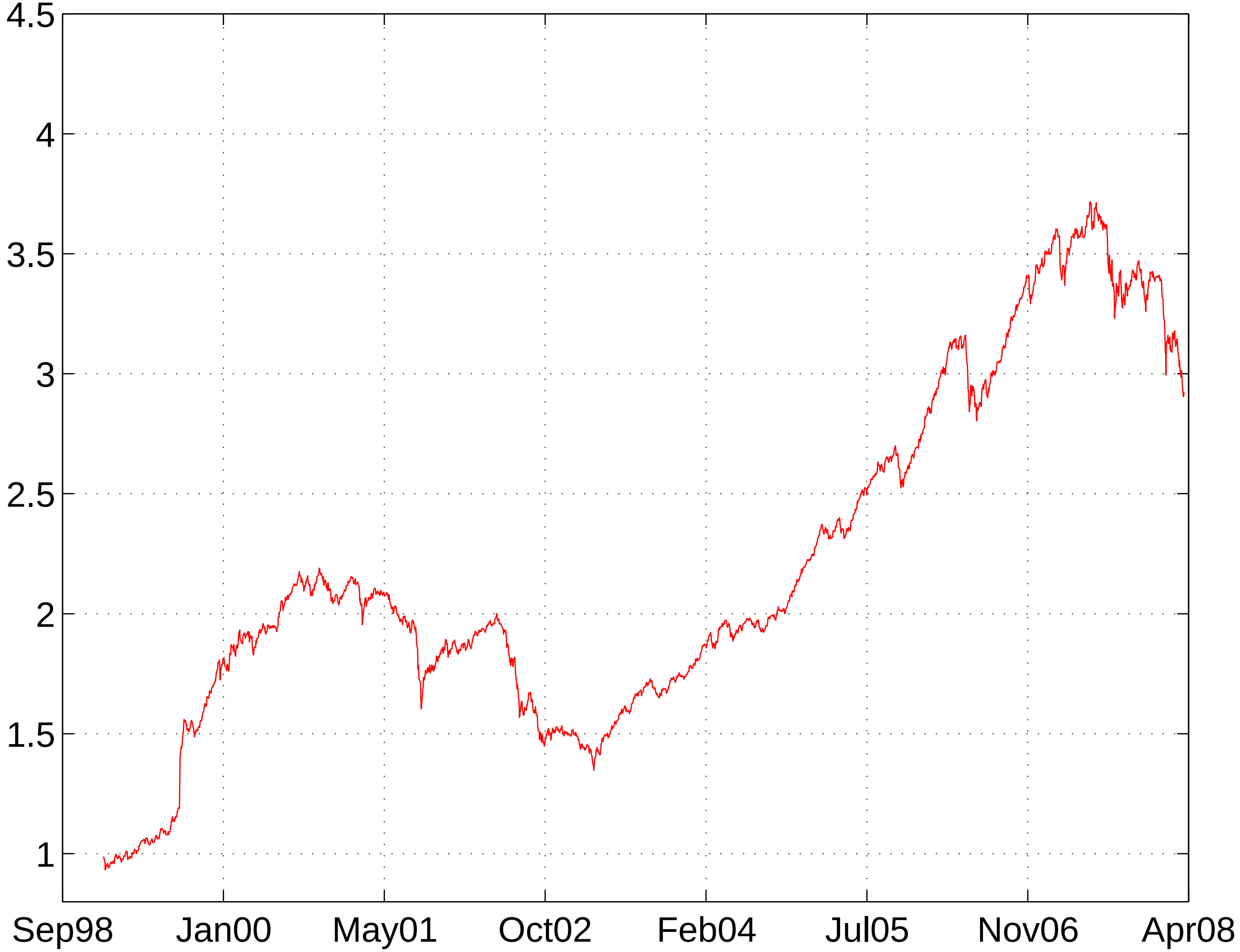
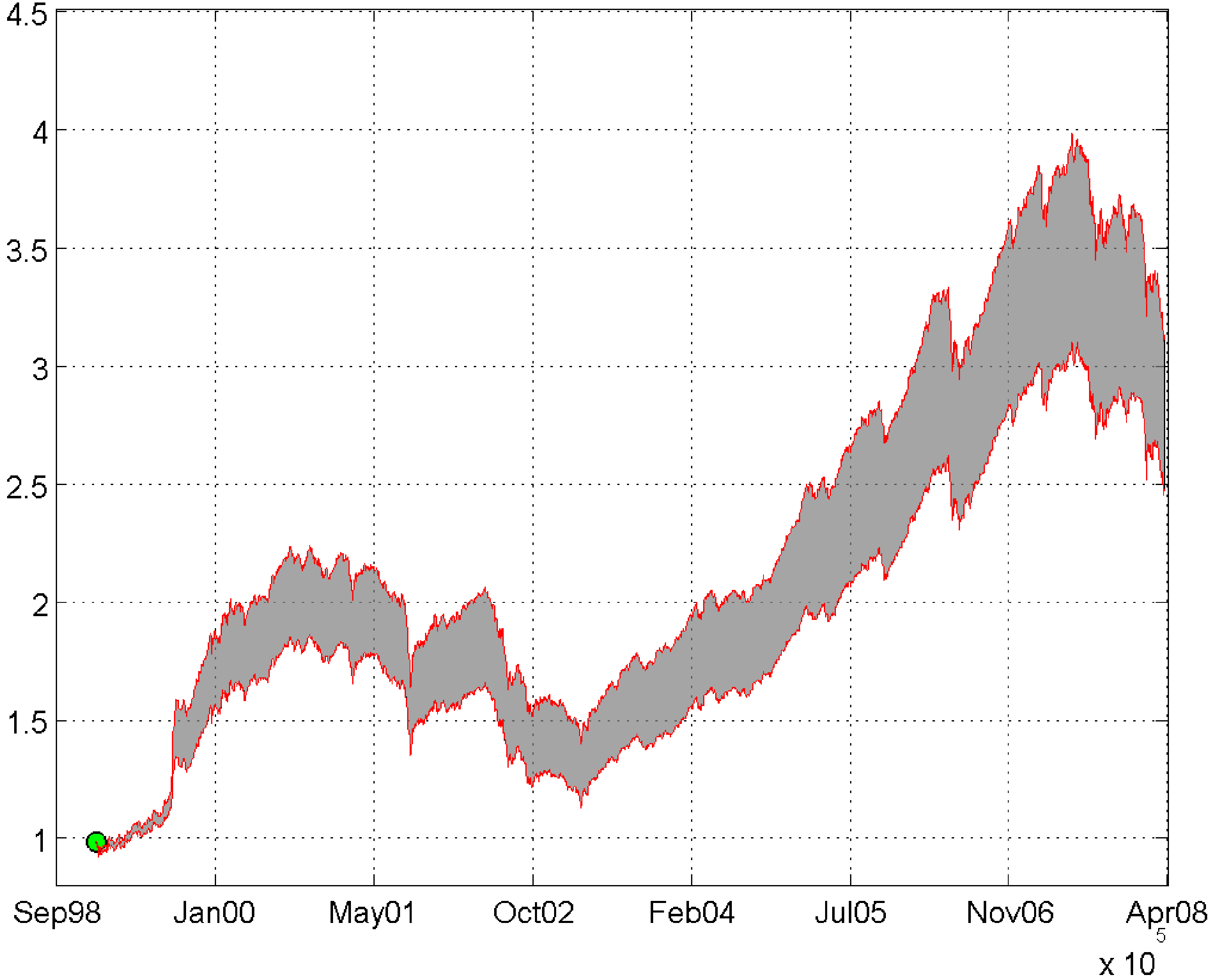
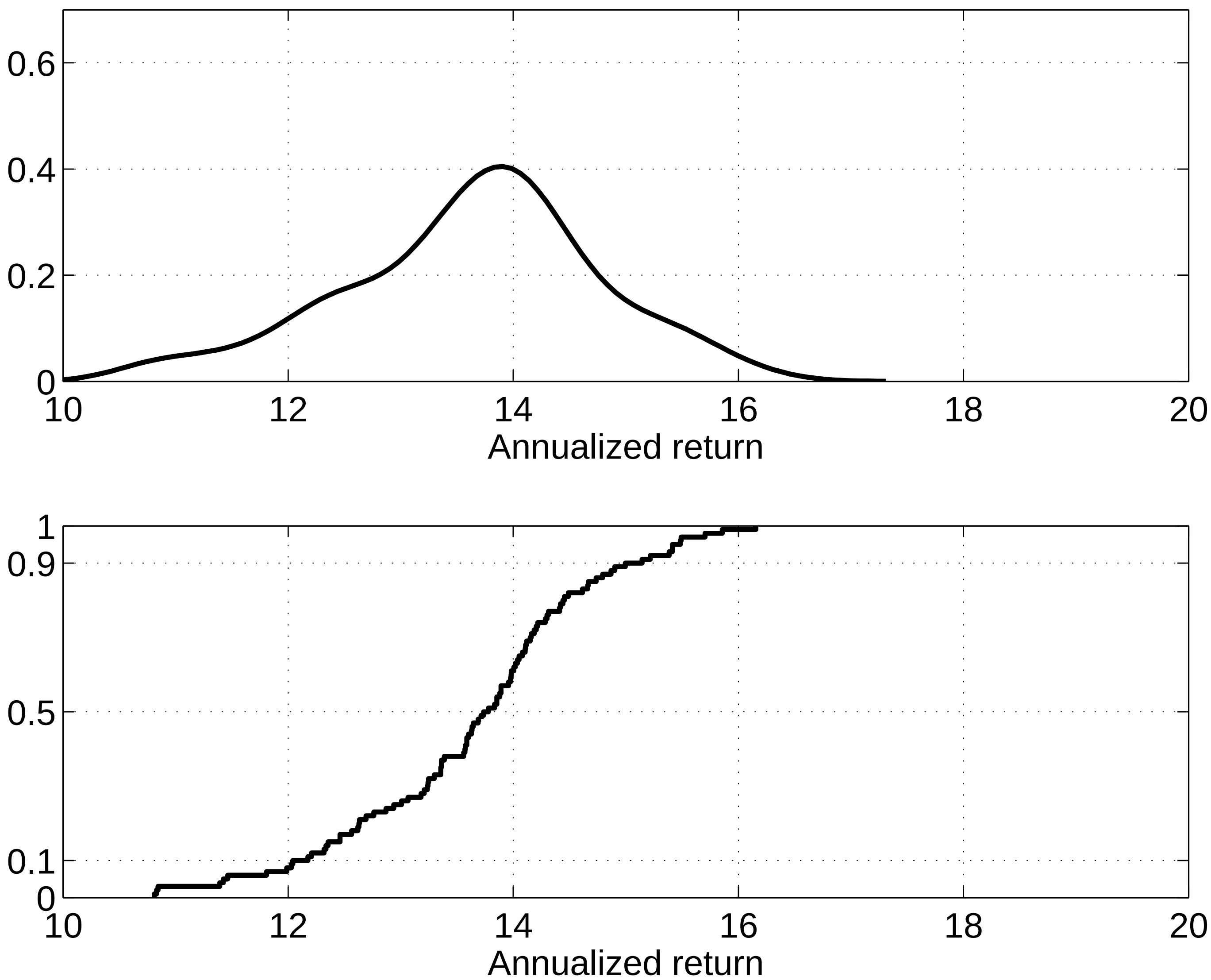


benchmark: MV long-only



benchmark: MV long-only



results VaR/Expected Shortfall

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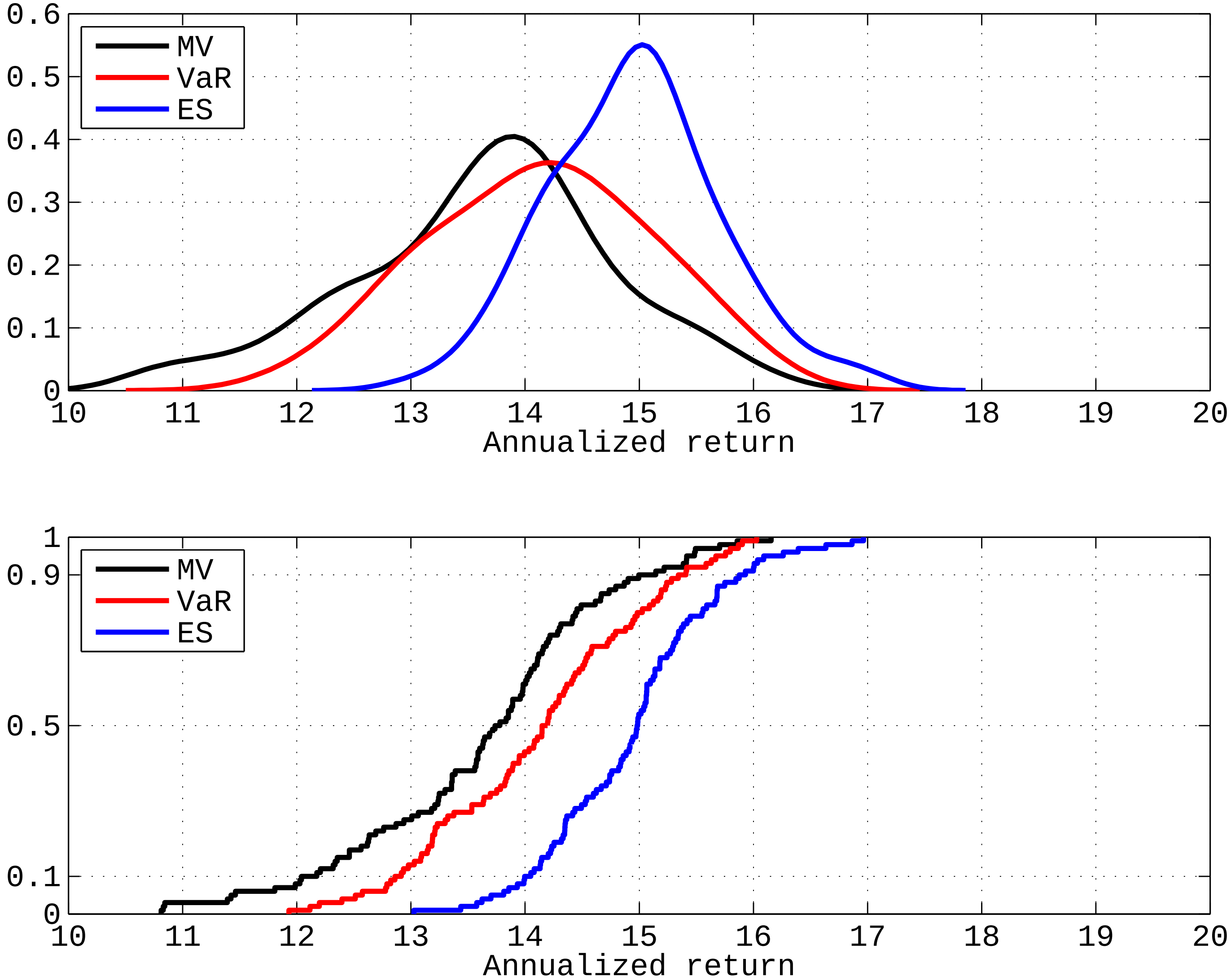
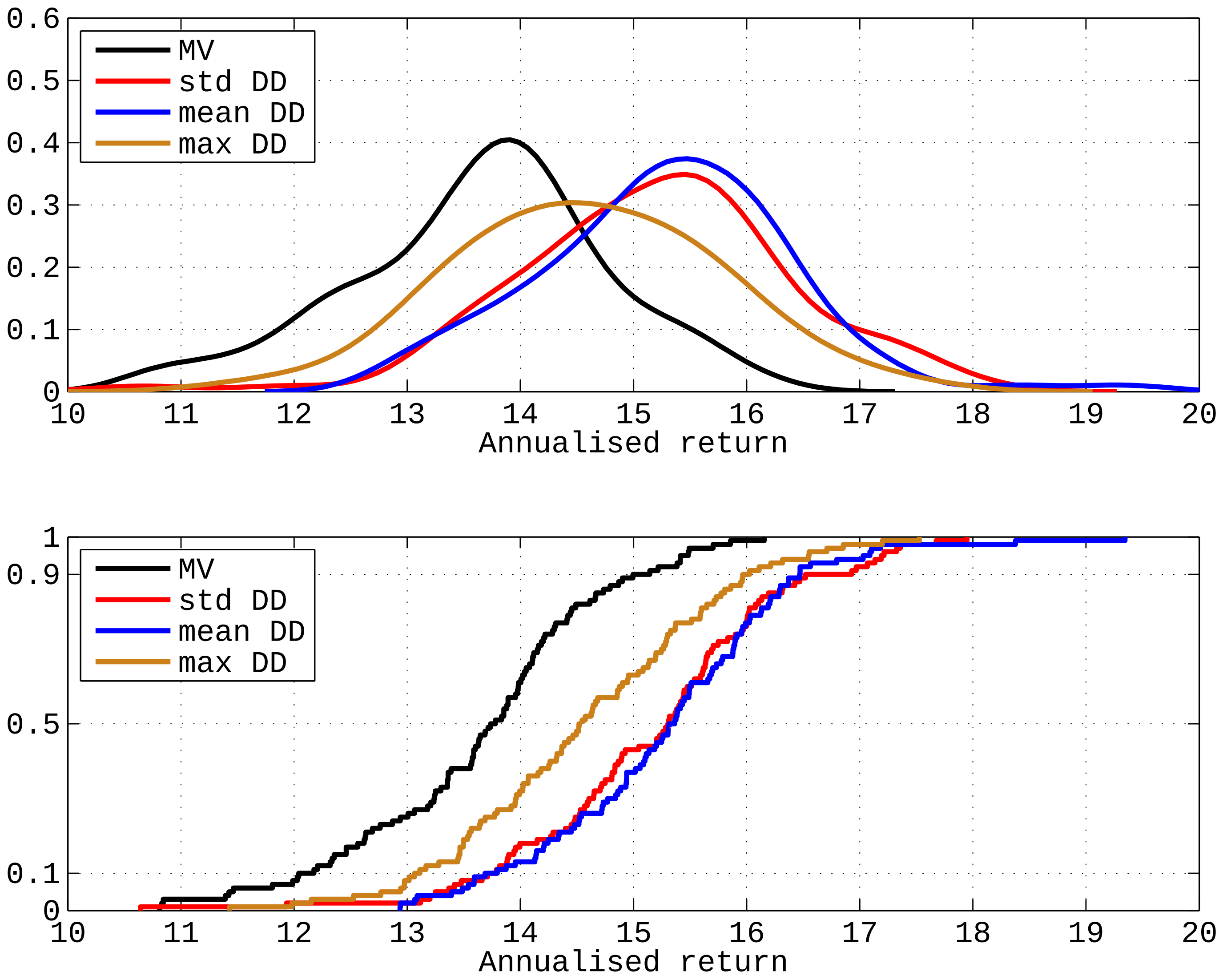


results drawdowns

results drawdowns



results drawdowns

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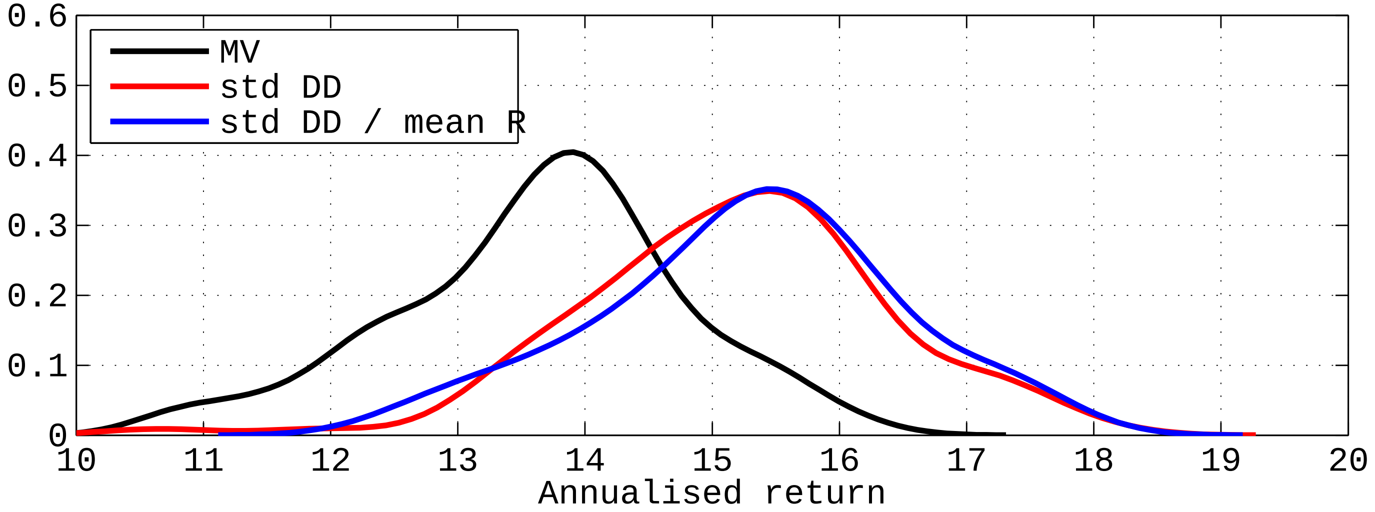
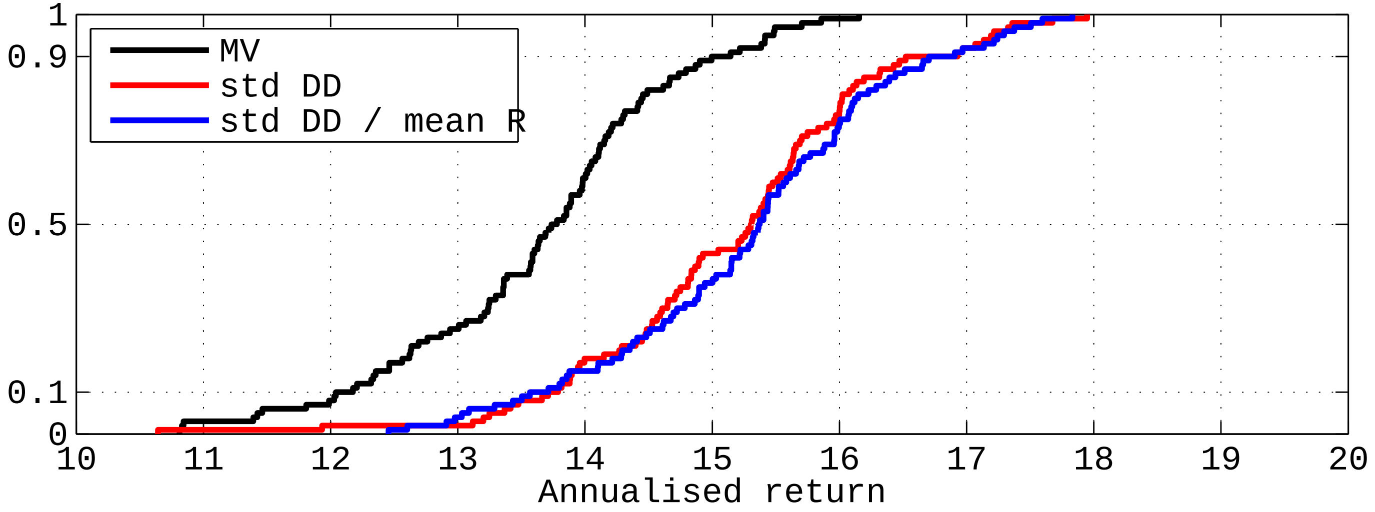
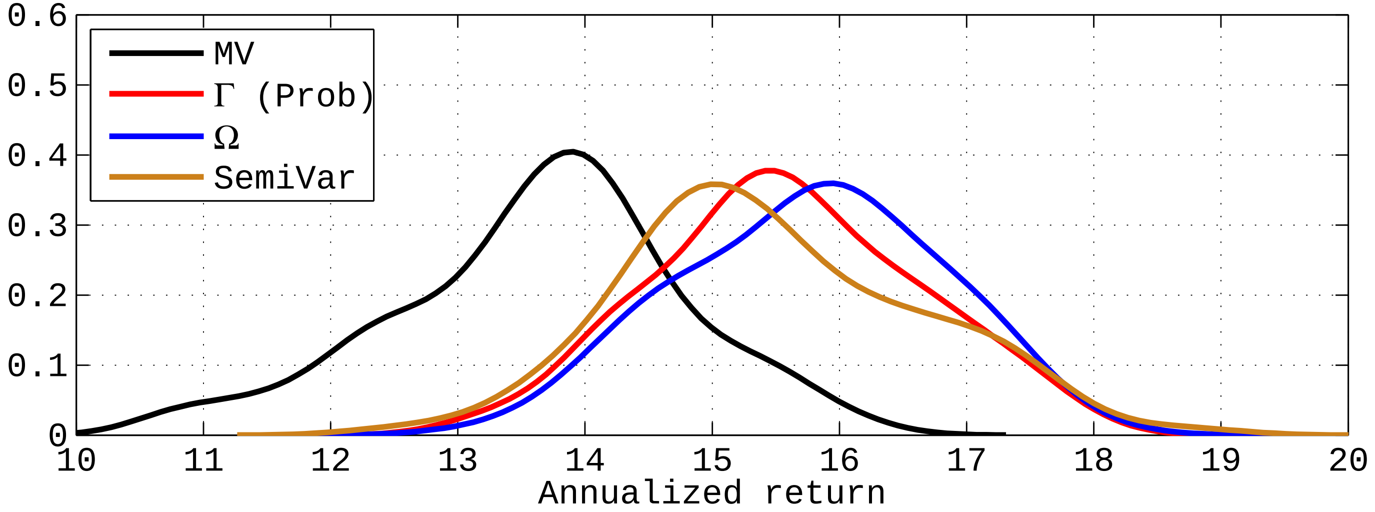
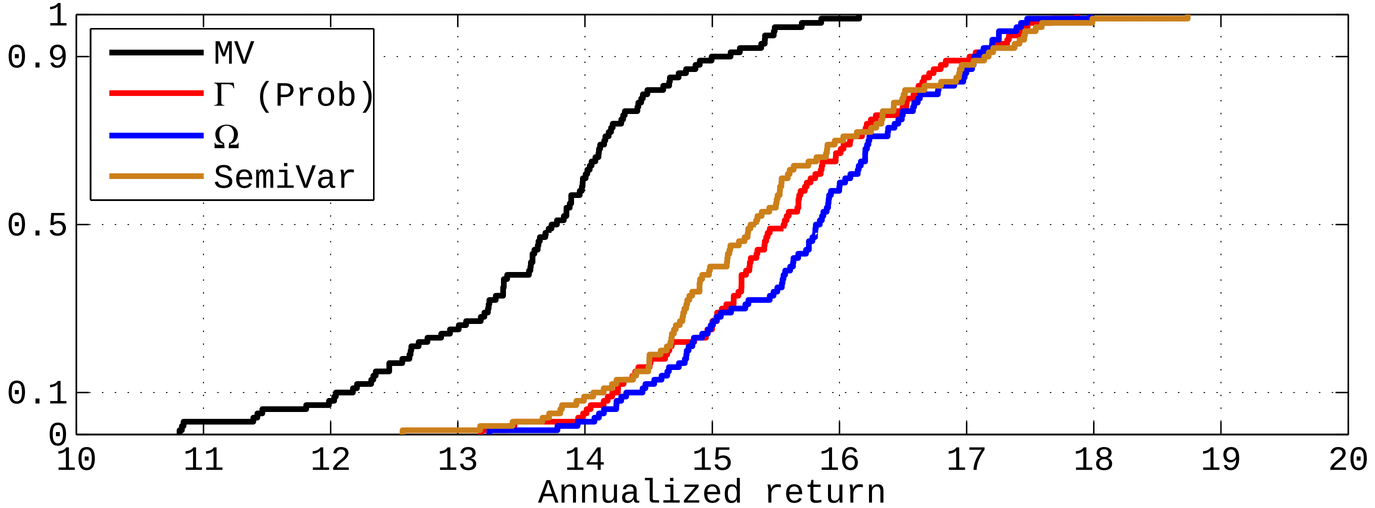




results partial moments

results partial moments





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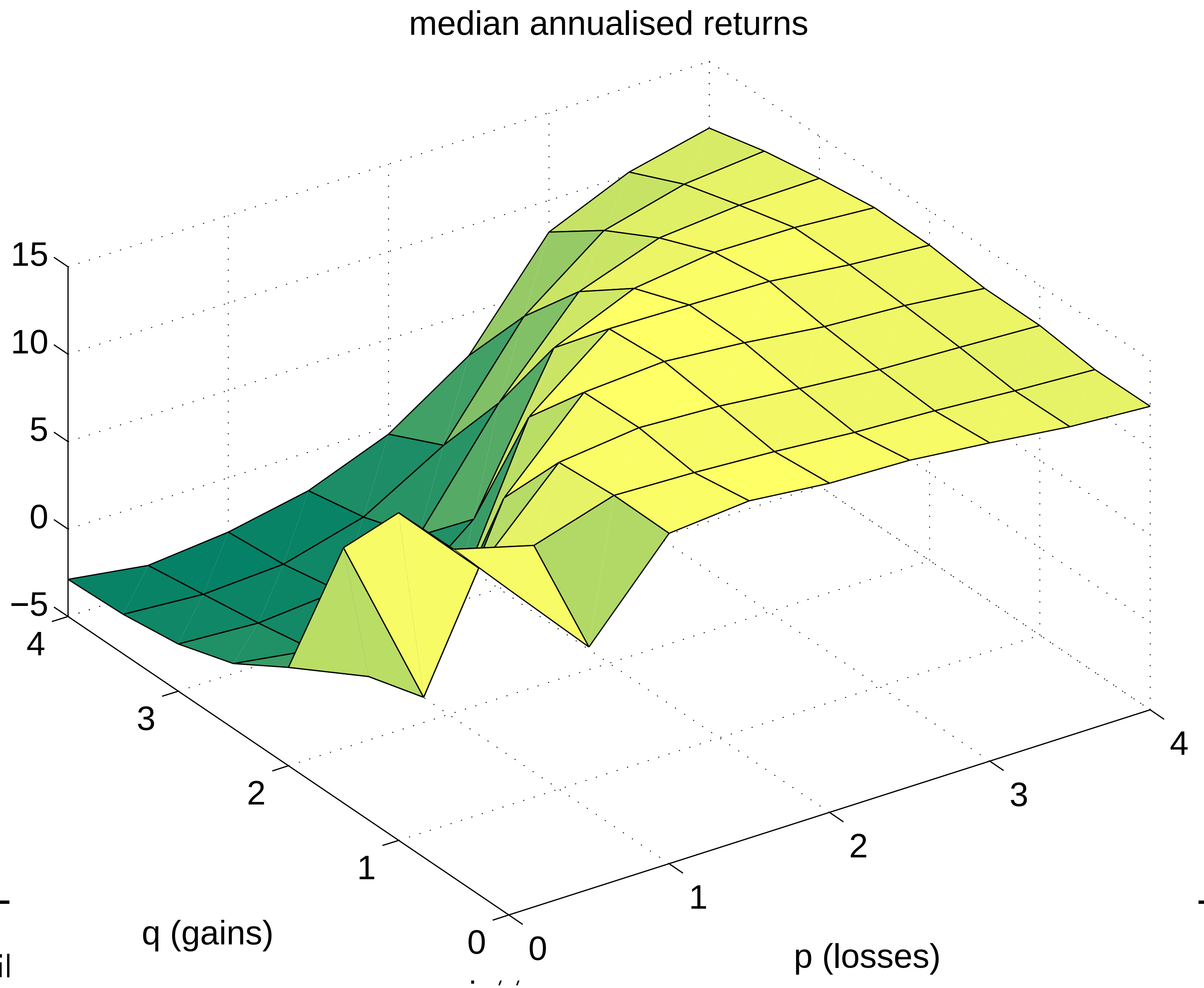

scenarios vs historical data

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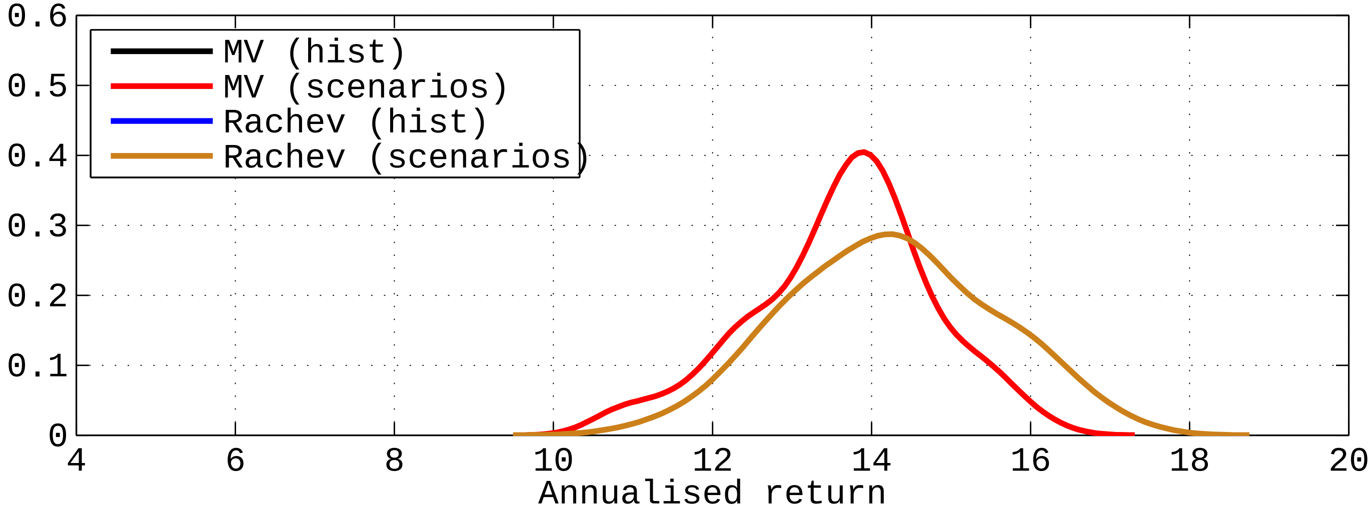

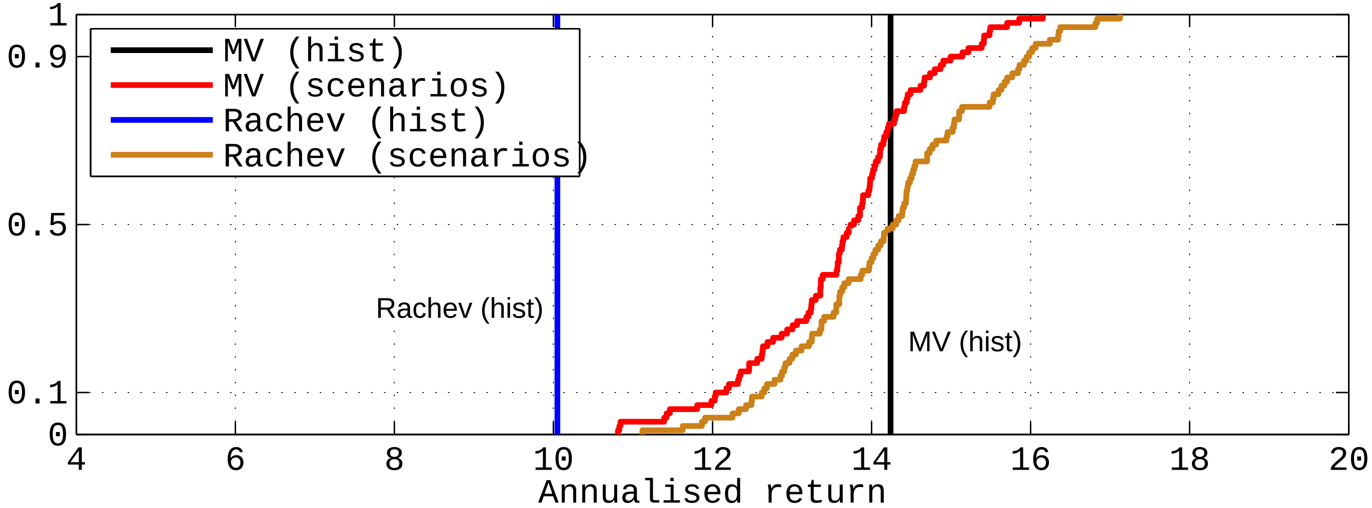

when is a solution 'optimal enough'?

when is a solution ‘optimal enough’?

$\longrightarrow$ characterise solution by objective function value

when is a solution ‘optimal enough’?

$\rightarrow$ characterise solution by objective function value
$\rightarrow$ run large number of optimisations $\rightarrow$ compare distribution of solutions

in-sample convergence

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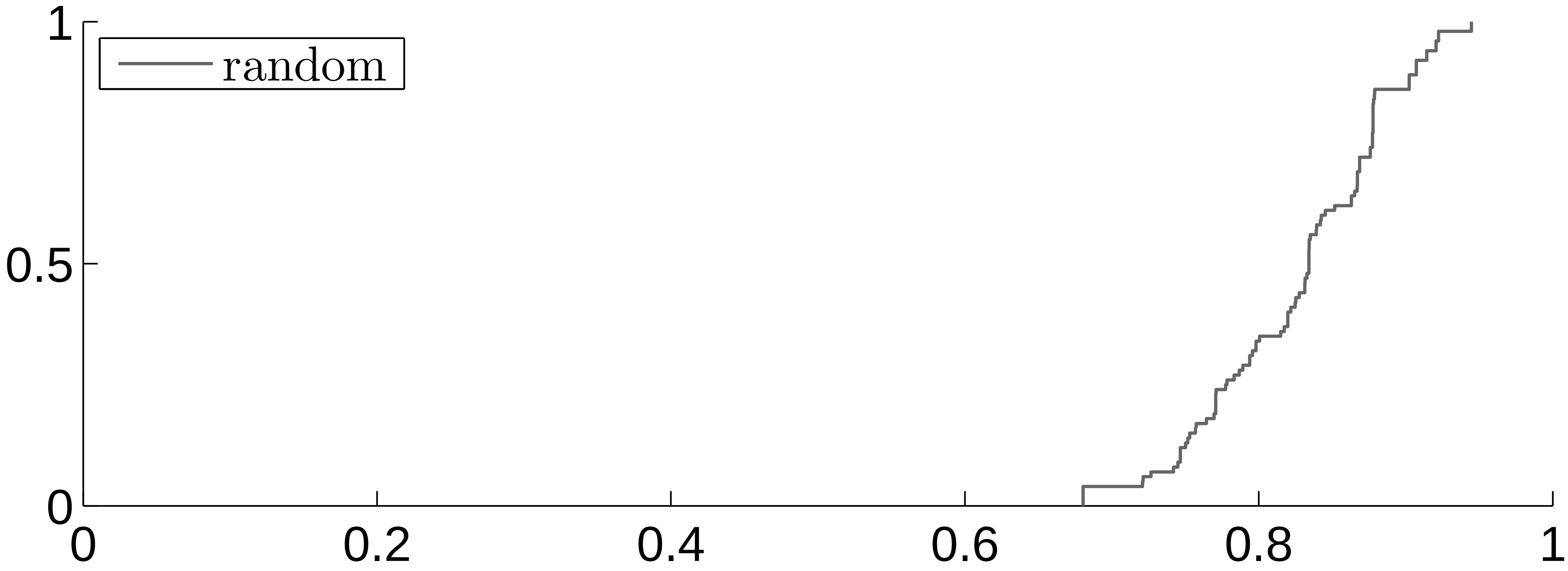
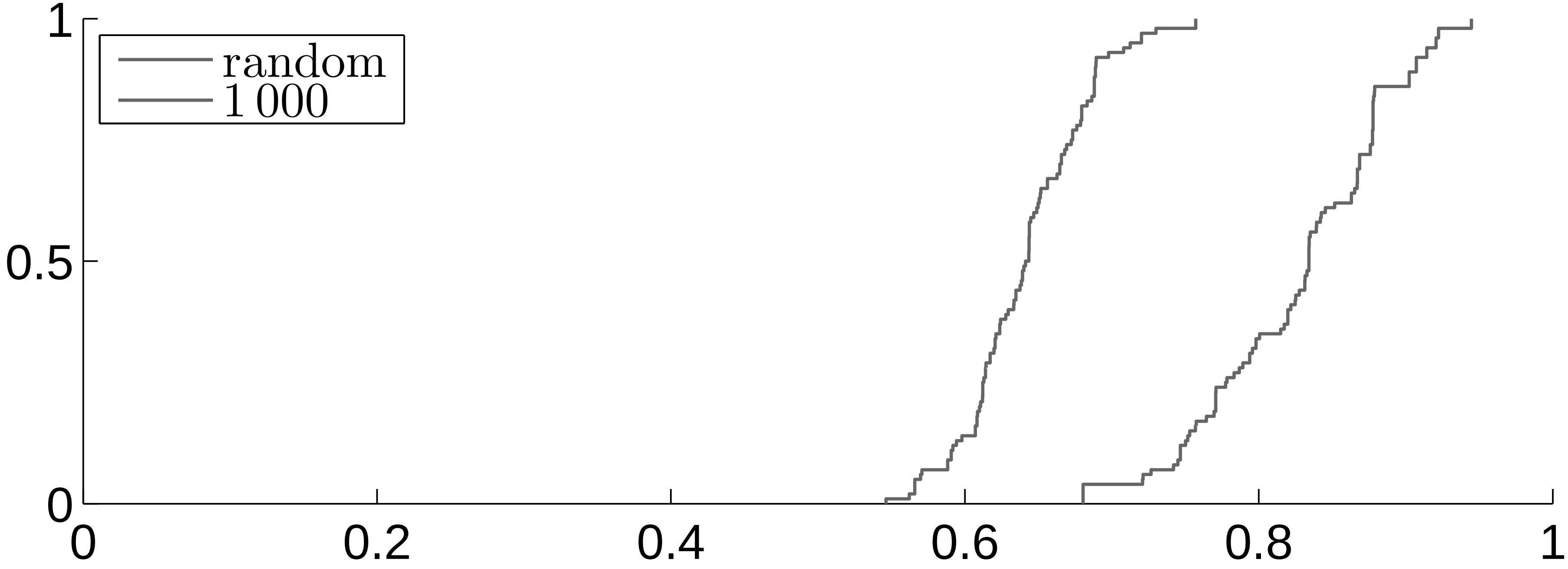
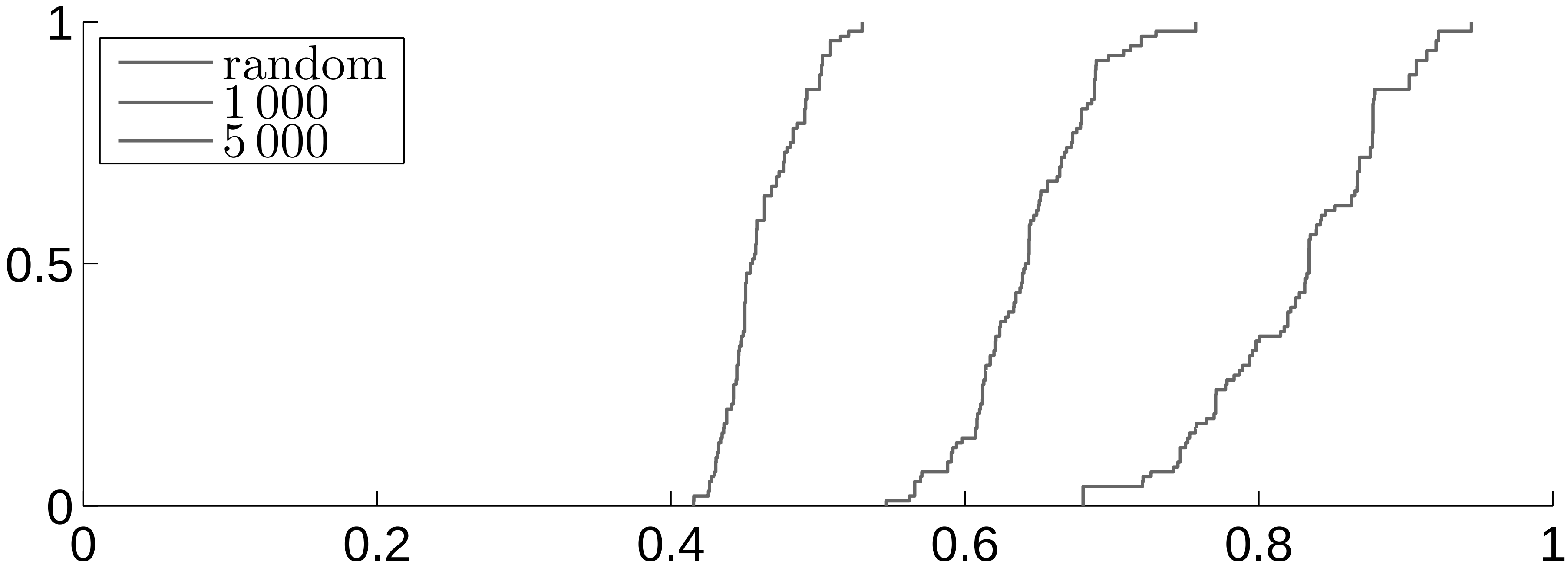


in-sample convergence



in-sample convergence



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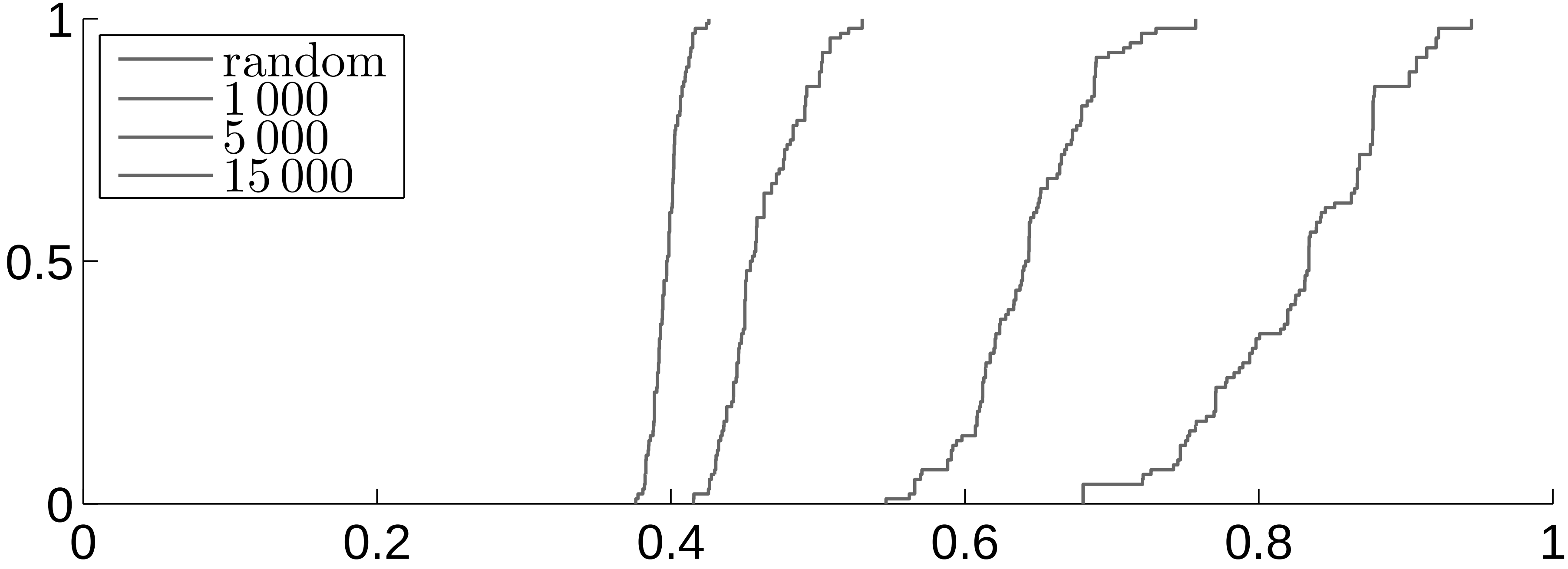

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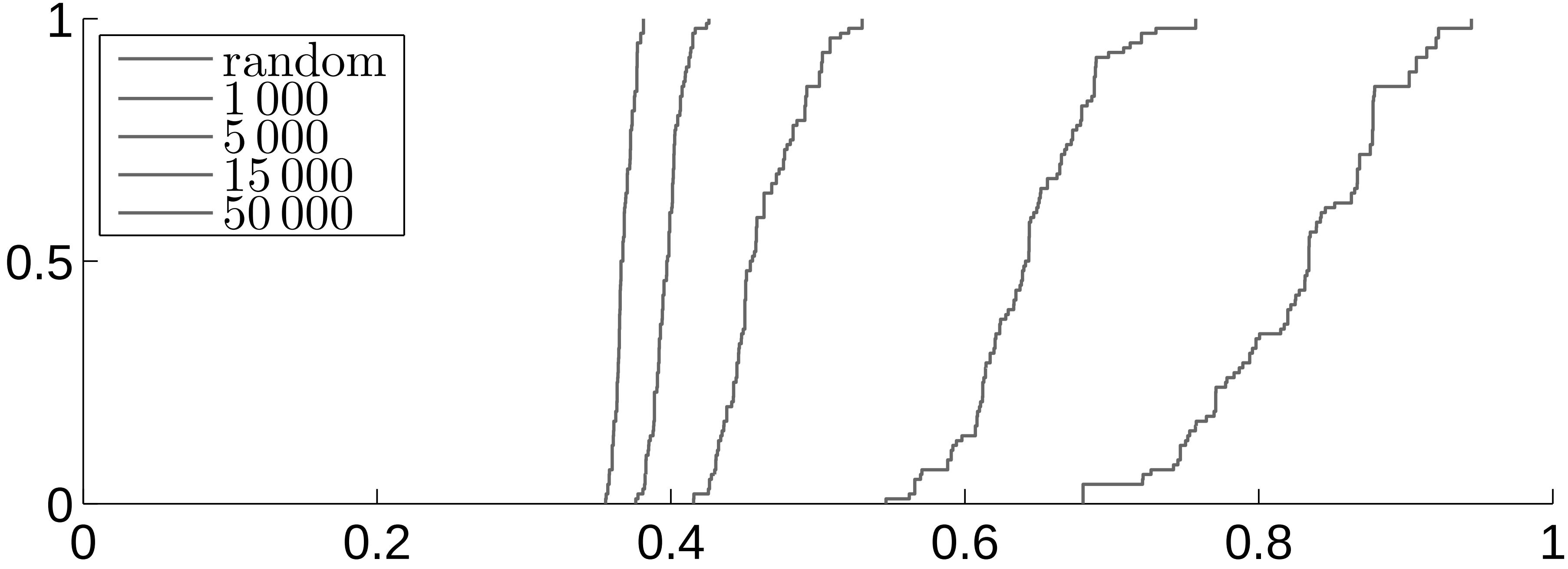

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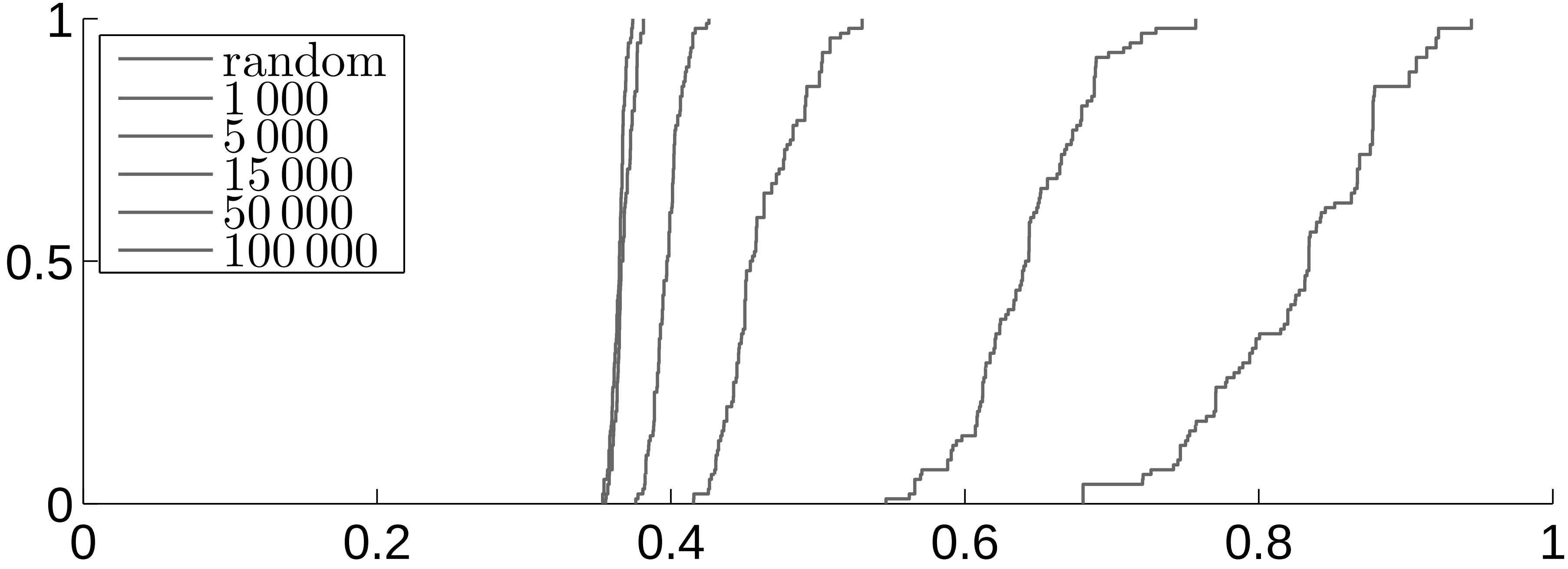
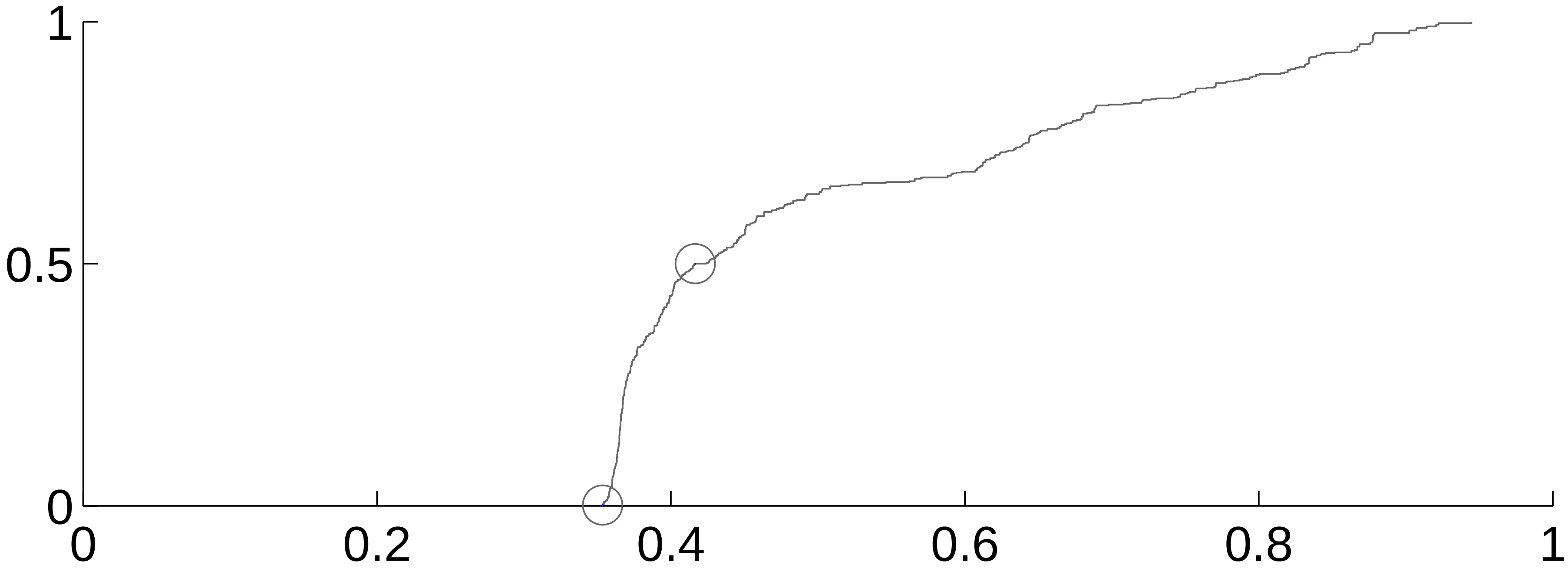
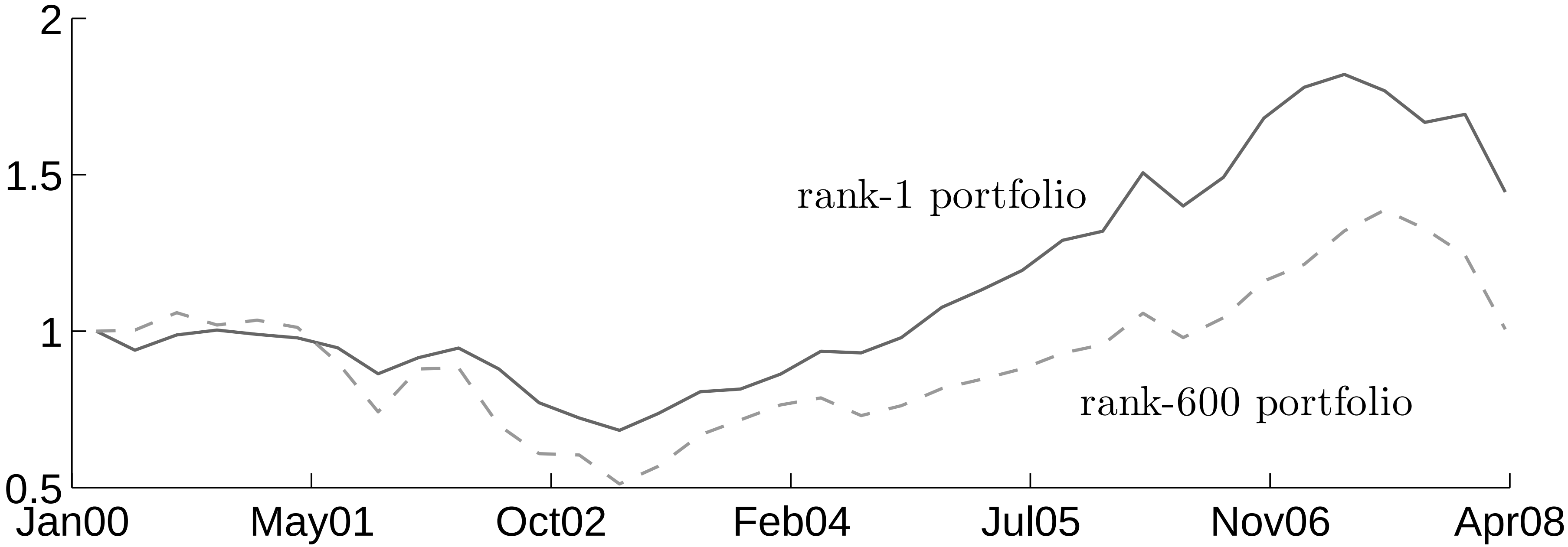


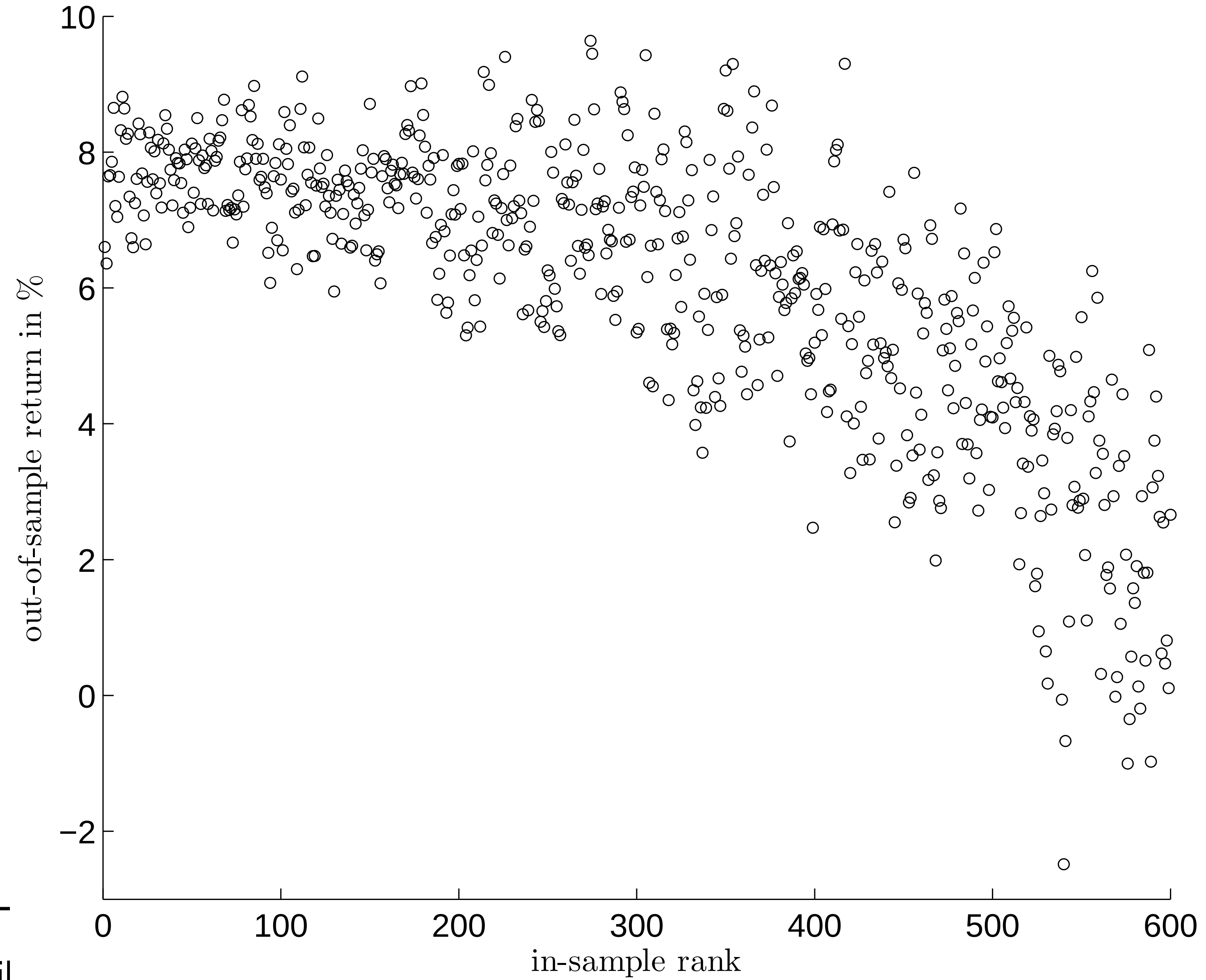
rank portfolios



rank portfolios, paths

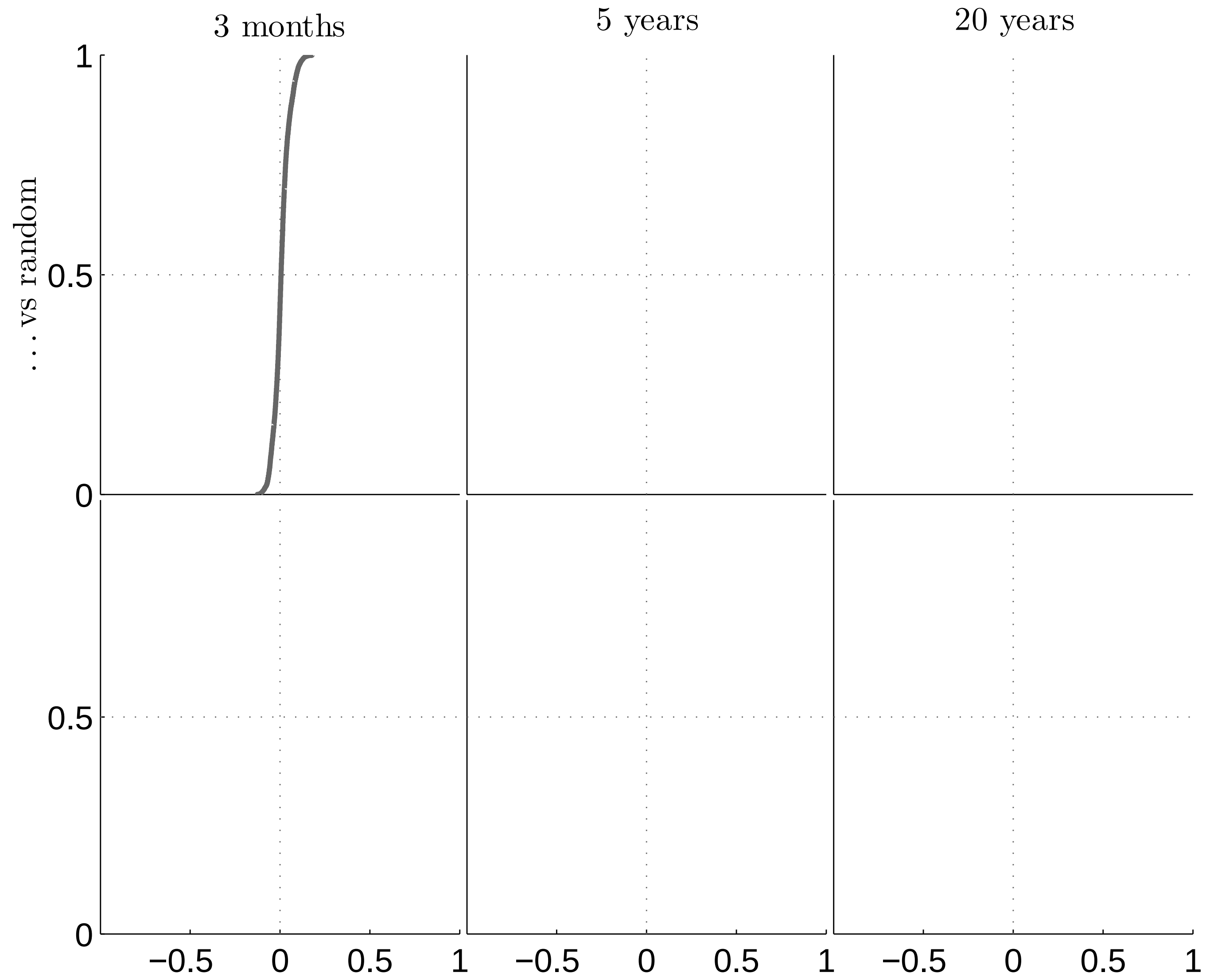


rank portfolios, final wealth

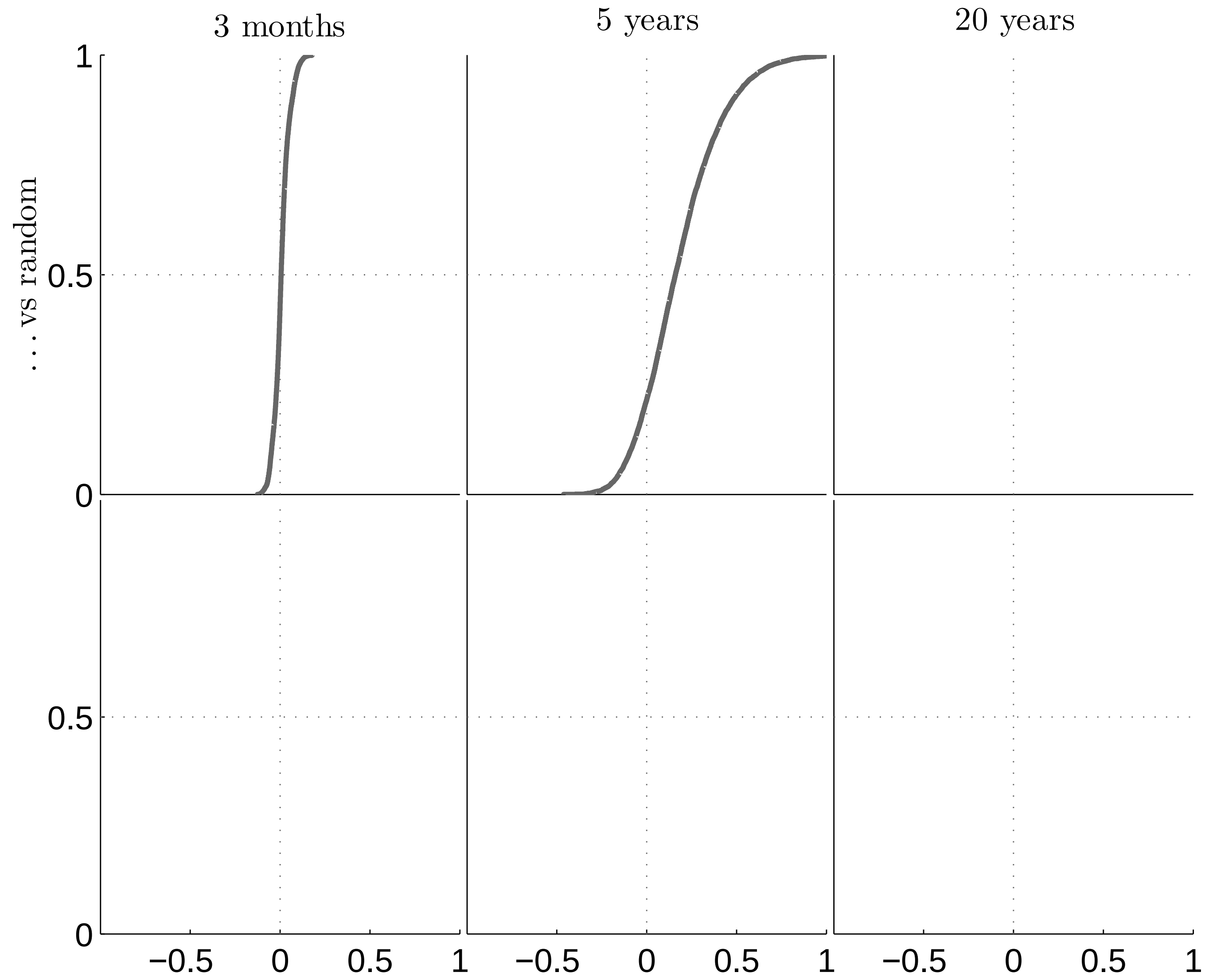


how many steps?

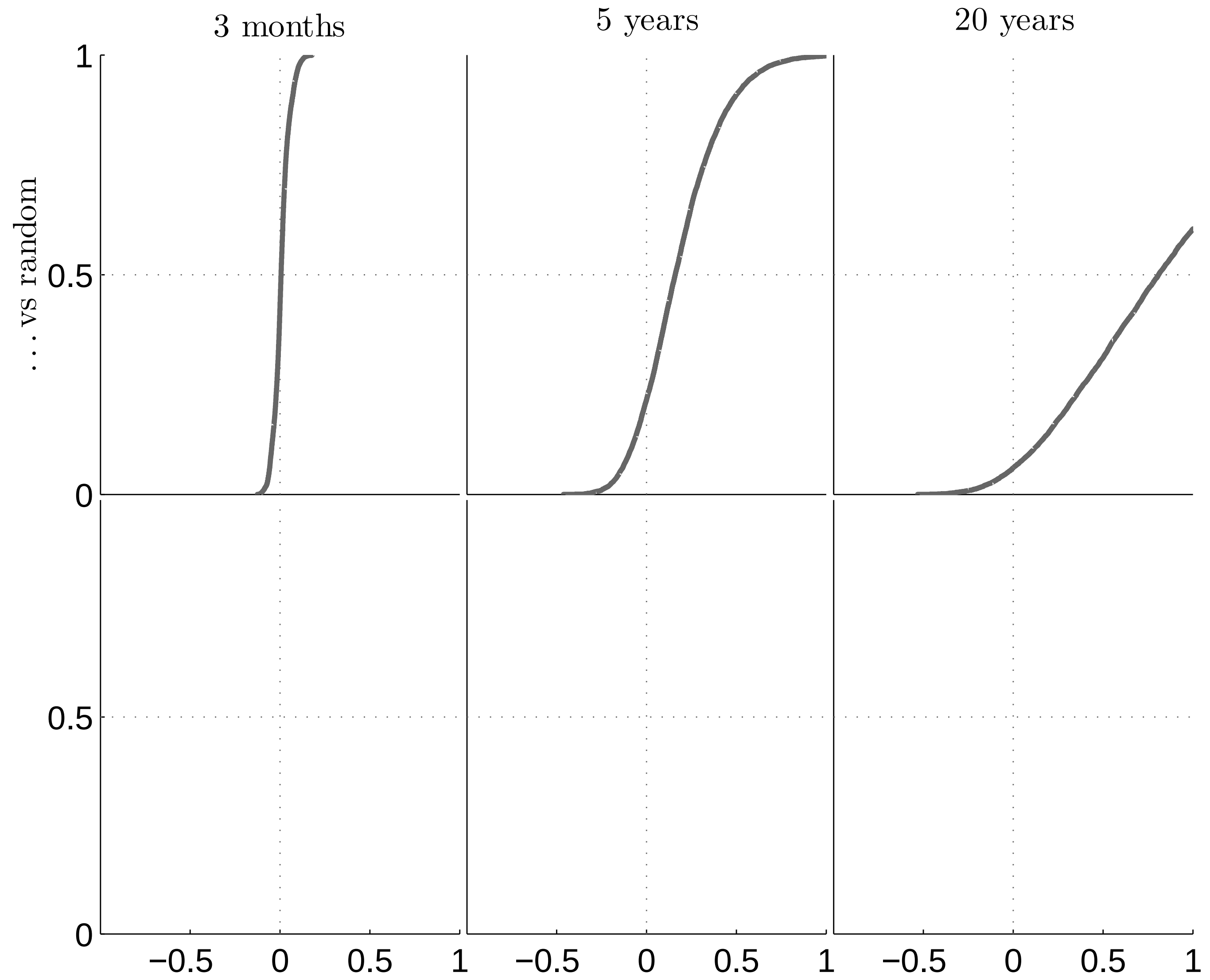
how many steps?



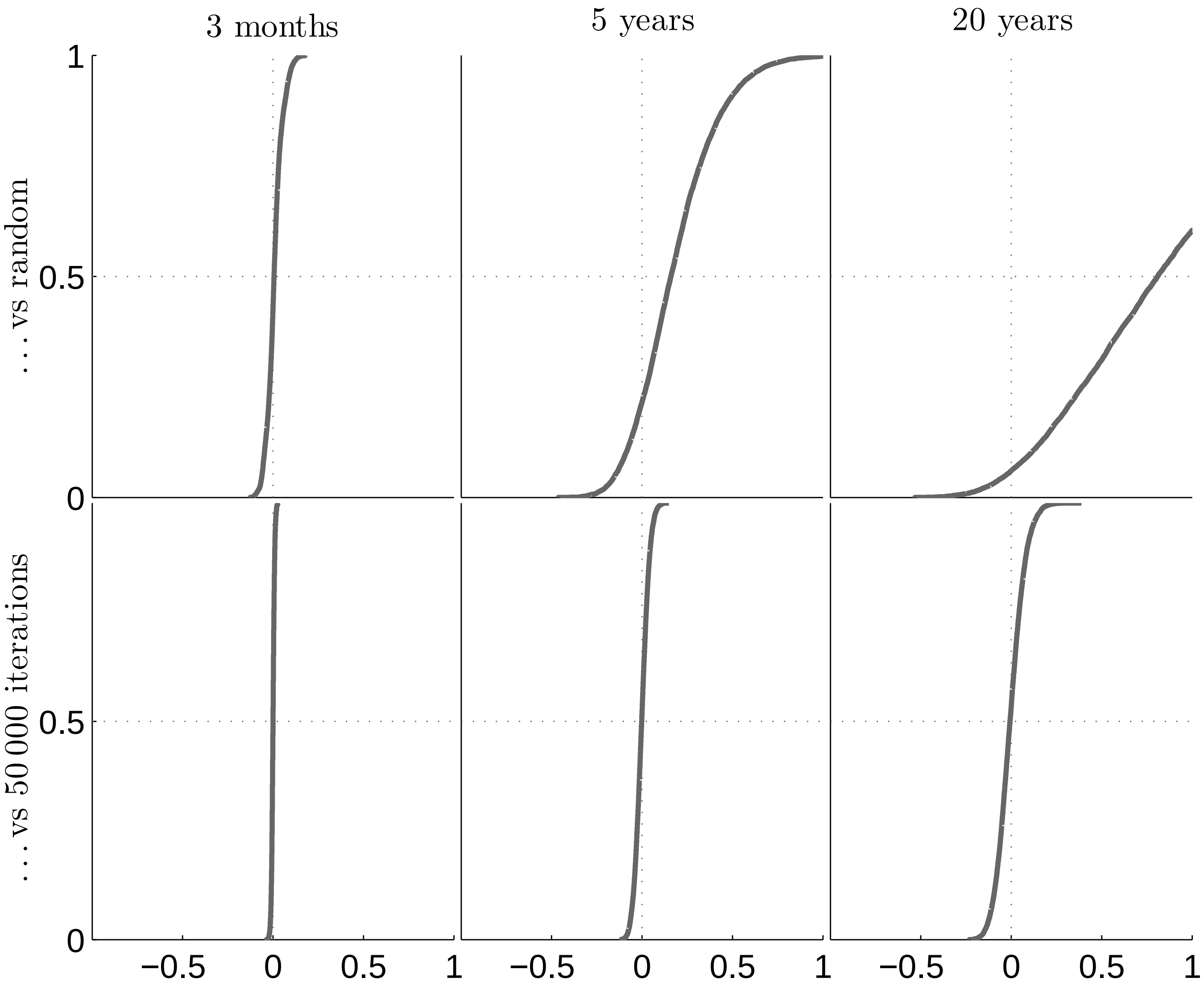
how many steps?



how many steps?



how many steps?



conclusions

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- alternative objective functions
  - optimisation more difficult, but manageable

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  - add value over mean–variance

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- alternative objective functions
  - optimisation more difficult, but manageable
  - add value over mean–variance
- problem very sensitive to data: ‘good’ solutions suffice

References

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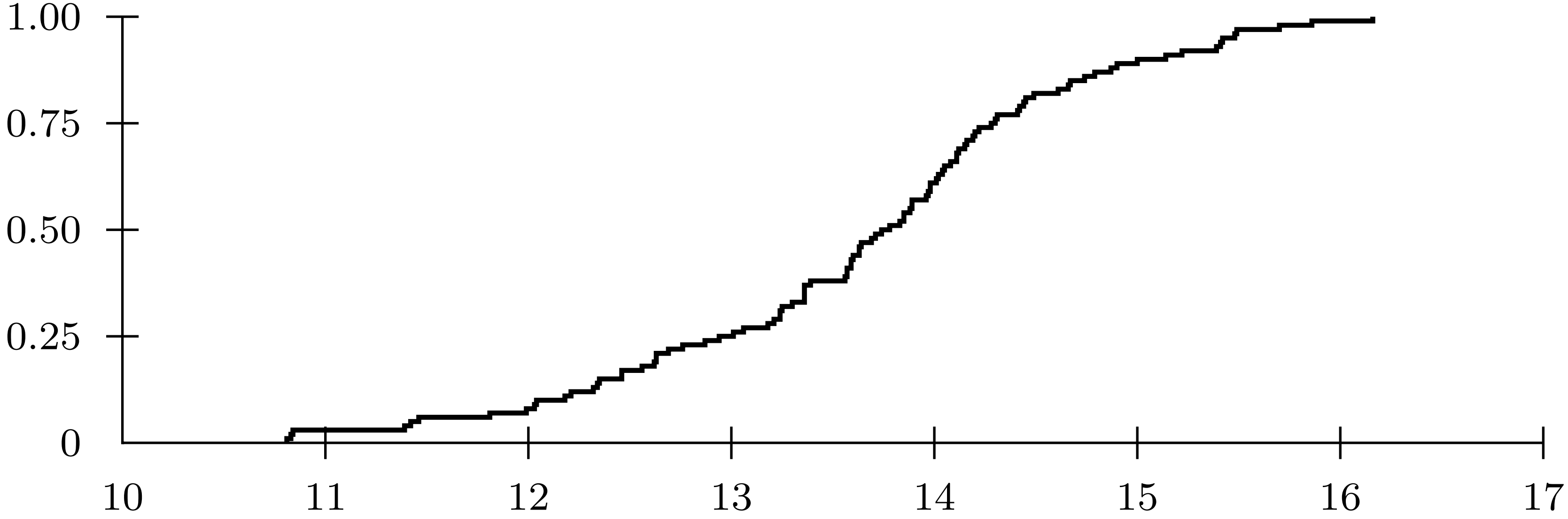
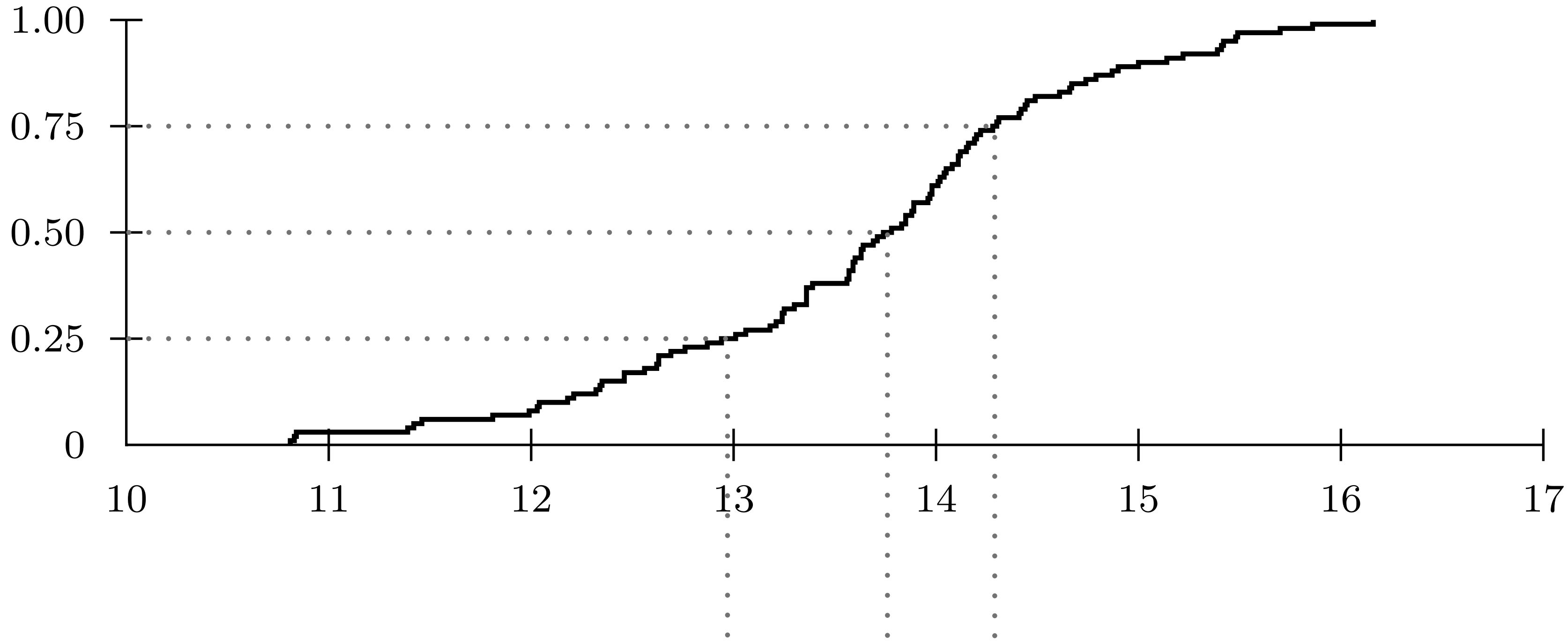
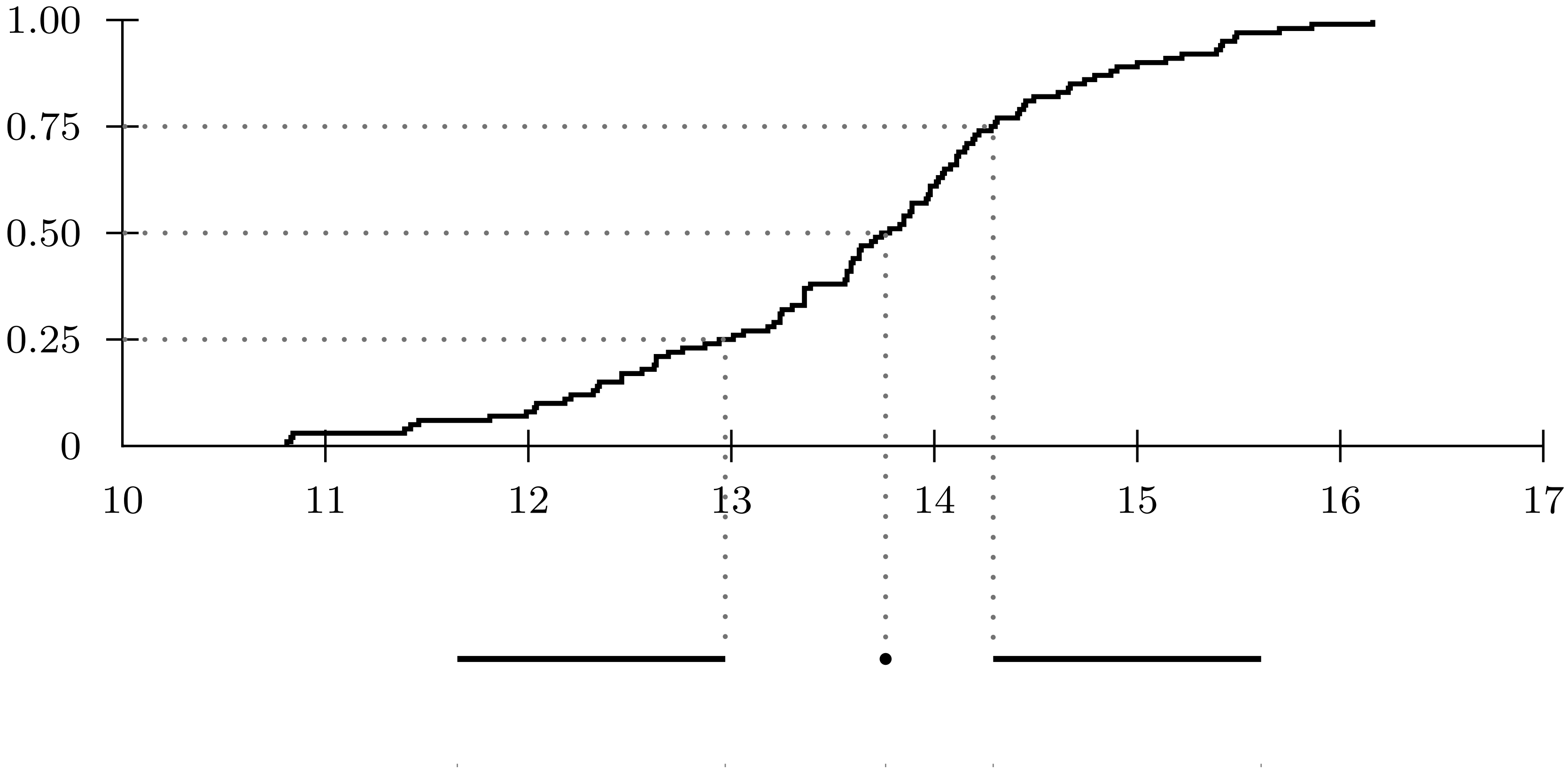
appendix: more results – MV portfolio



appendix: more results – MV portfolio



appendix: more results – MV portfolio



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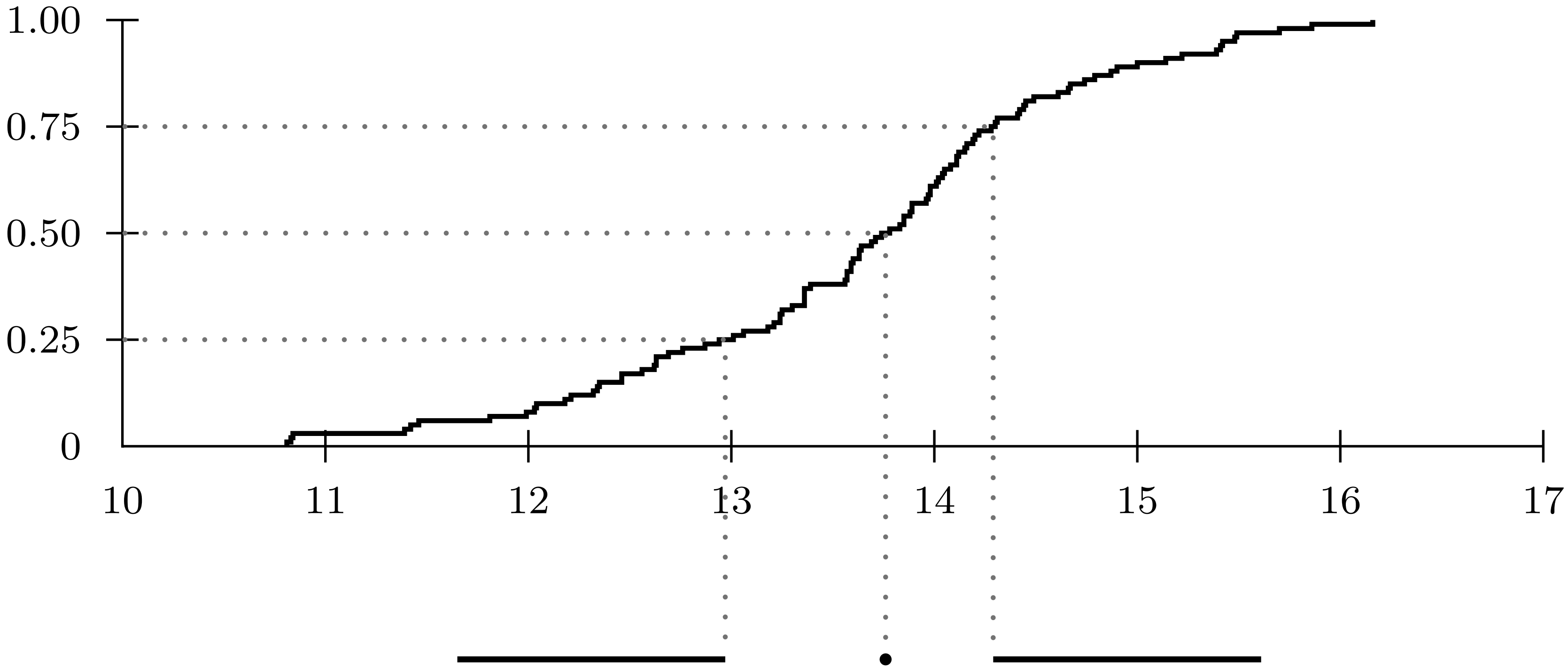
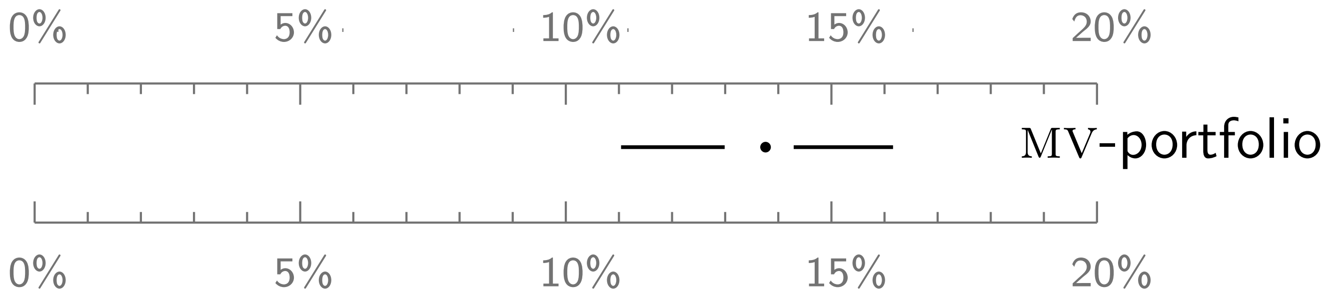


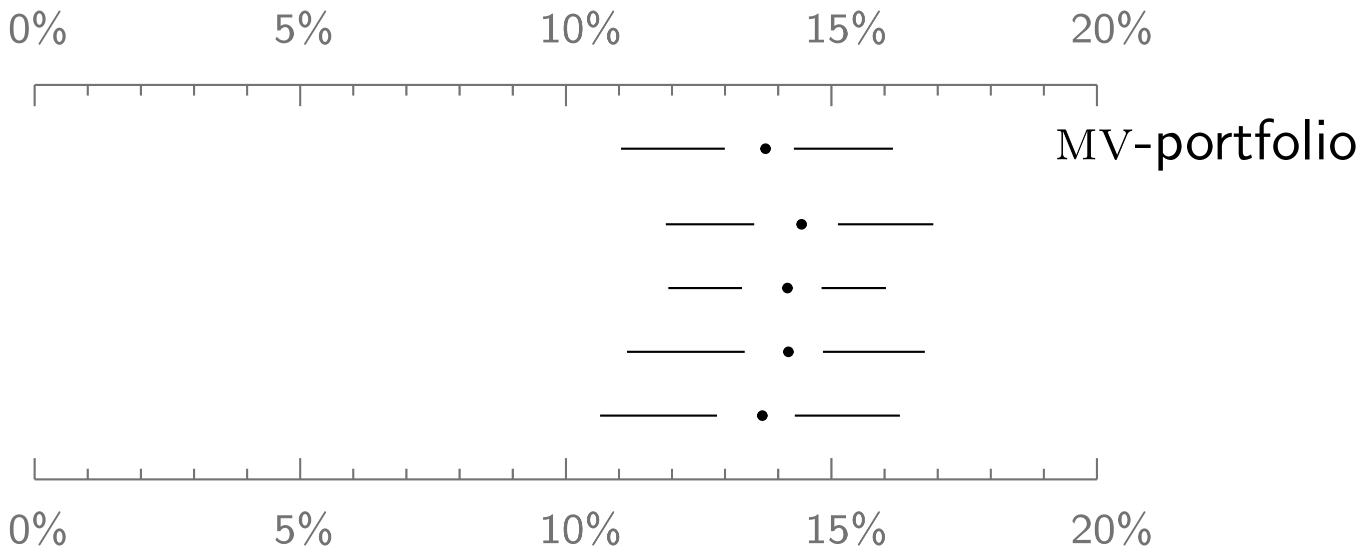
reward	risk	median
c	$\mathcal{M}_2$	13.76



results VaR, ES

results VaR, ES

reward	risk	median
c	$\mathcal{M}_2$	13.76
c	$\mathcal{Q}_1$	14.44
c	$\mathcal{Q}_5$	14.17
c	$\mathcal{Q}_{10}$	14.19
c	$\mathcal{Q}_{20}$	13.70



reward	risk	median
c	$\mathcal{M}_2$	13.76
c	$\mathcal{Q}_1$	14.44
c	$\mathcal{Q}_5$	14.17
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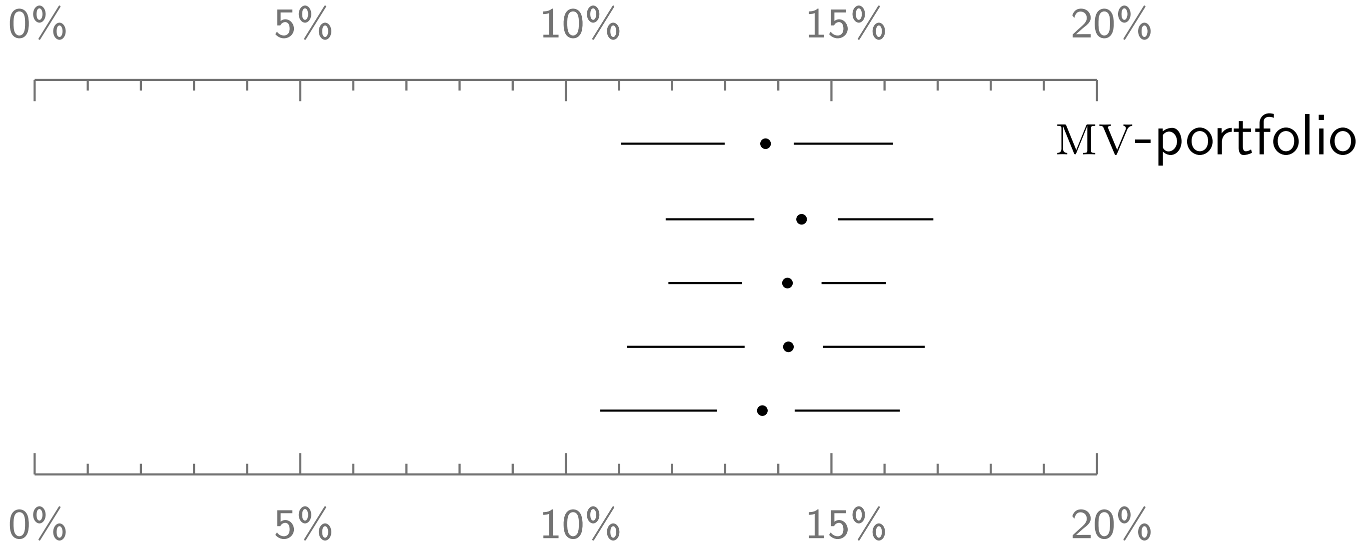
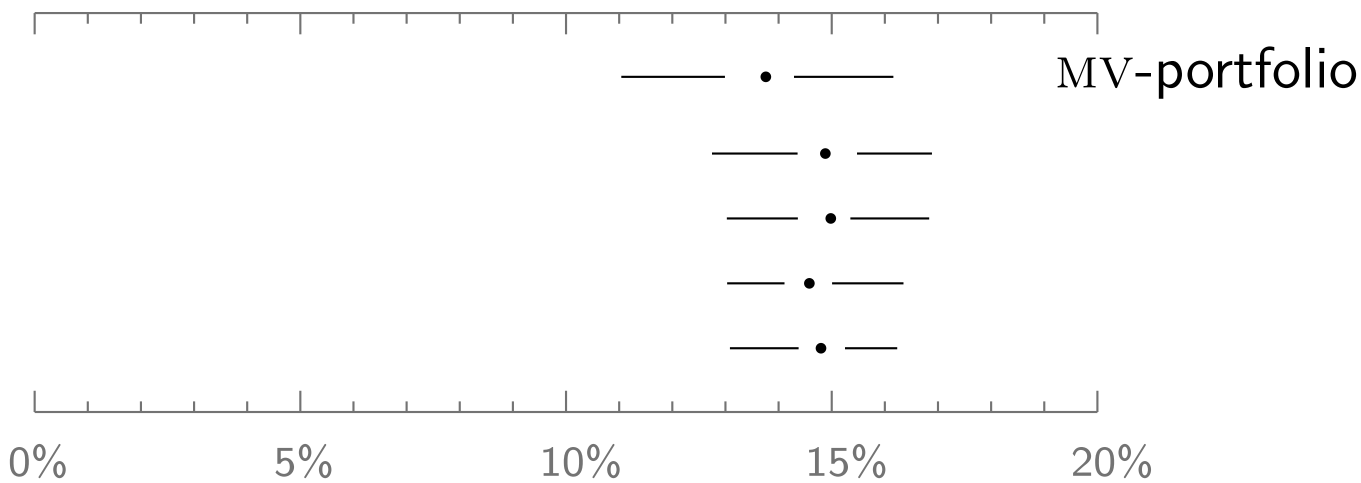
c	$\mathcal{Q}_{20}$	13.70



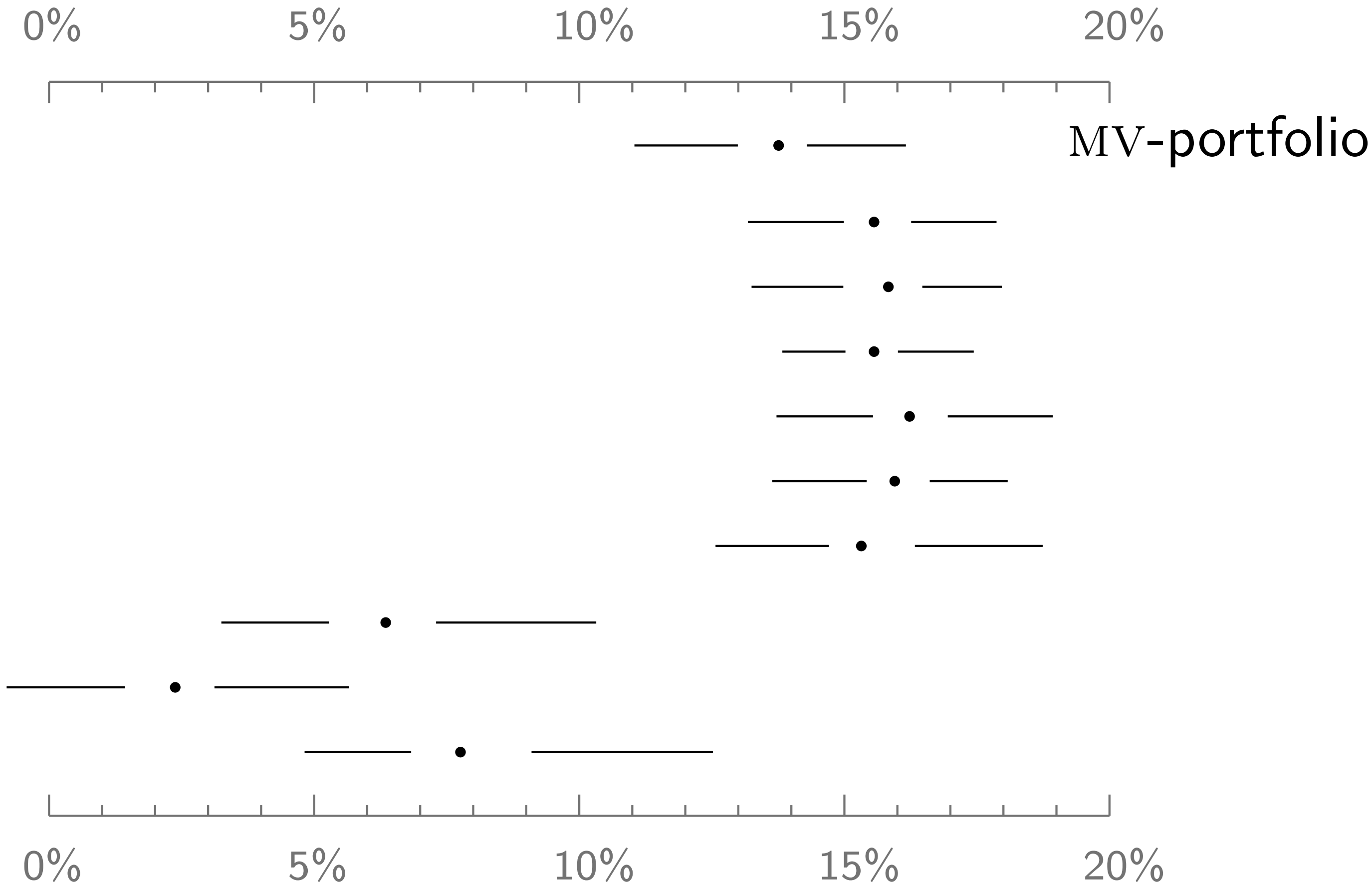
reward	risk	median
c	$\mathcal{M}_2$	13.76
c	$\mathcal{C}_1^-(\mathcal{Q}_1)$	14.88
c	$\mathcal{C}_1^-(\mathcal{Q}_5)$	14.98
c	$\mathcal{C}_1^-(\mathcal{Q}_{10})$	14.58
c	$\mathcal{C}_1^-(\mathcal{Q}_{20})$	14.80



results partial moments

results partial moments

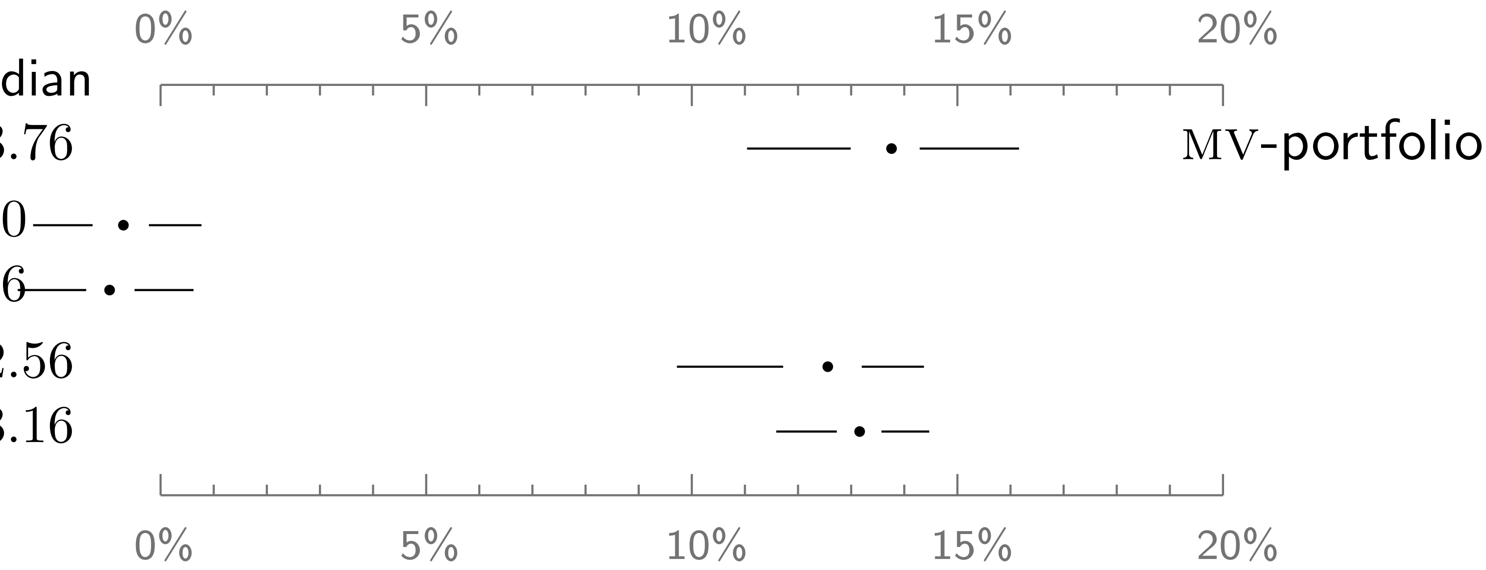
reward	risk	median
c	$\mathcal{M}_2$	13.76
$\mathcal{P}_0^+$	$\mathcal{P}_0^-$	15.56
$\mathcal{P}_1^+$	$\mathcal{P}_1^-$	15.83
$\mathcal{P}_1^+$	$\mathcal{P}_2^-$	15.56
$\mathcal{P}_{1.5}^+$	$\mathcal{P}_{1.5}^-$	16.23
$\mathcal{P}_{1.5}^+$	$\mathcal{P}_2^-$	15.95
$\mathcal{P}_2^+$	$\mathcal{P}_2^-$	15.32
$\mathcal{P}_3^+$	$\mathcal{P}_2^-$	6.35
$\mathcal{P}_4^+$	$\mathcal{P}_2^-$	2.38
$\mathcal{P}_4^+$	$\mathcal{P}_3^-$	7.76



results partial moments

results partial moments

reward	risk	median
c	$\mathcal{M}_2$	13.76
$\mathcal{P}_2^+$	c	-0.70
$\mathcal{P}_3^+$	c	-0.96
c	$\mathcal{P}_2^+$	12.56
c	$\mathcal{P}_3^+$	13.16



results drawdowns

results drawdowns

reward	risk	median
c	$\mathcal{M}_2$	13.76
c	$\mathcal{D}_{\text{mean}}$	15.34
$\mathcal{M}_1$	$\mathcal{D}_{\text{mean}}$	15.56
c	$\mathcal{D}_{\max}$	14.53
$\mathcal{M}_1$	$\mathcal{D}_{\max}$	14.93
c	$\mathcal{D}_{\text{std}}$	15.31
$\mathcal{M}_1$	$\mathcal{D}_{\text{std}}$	15.37